Vectors (EDR-Momentum): Triangulating Momentum

Angular Collision Alternative to V3 & V4 Requirement of Linear Momentum

In the beginning....

Five modules in **REC-TEC** offer an alternative to requiring **V3** and **V4** speeds to reach a collision solution.

Digital Momentum, **Linear Momentum**, **PDOF/360LM** in the **360 LM** mode, **CrushV** Damage Analysis, and **Vectors (EDR - Momentum)** now can use the Longitudinal and Lateral DeltaV (or PDOF) of one vehicle to solve for DV1, DV2, V1, V2, V3, and V4. This method uses the Law of Sines, the Law of Cosines and Newton's Third Law of Motion. In Digital and Linear Momentum, and 360 LM the program solves for V3 and V4 using this method and uses those values to solve the Conservation of Linear Momentum (COLM) equations. In **Vectors (EDR-Momentum)** and **CrushV** it uses this method to solve for all of the unknowns.

Longitudinal DeltaV (X) - (CDR download)
Lateral DeltaV (Y) - (CDR download)
or
PDOF1 (Degrees) - User supplied

Angle 2 (Degrees)
Angle 3 (Degrees)
Angle 4 (Degrees)

Slip Angle (Degrees) [Optional] – Angle between the Longitudinal Axis of Vehicle 1 and its Post-Impact direction of travel
Weight (1)
Weight (2)

With Event Data Recorder (EDR) downloads becoming increasingly available, this new methodology offers a valuable alternative to the V3 and V4 requirement, one not always as reliable or accurate as desired. It also offers a convenient cross check when V3 and V4 are available.

Formulae:

DV1 = DV1X / Cos(PDOF1 + Abs(SlipAngle))
DV2 = DV1 * (W1 / W2)

AngleC = 180 - (A3 + PDOF1)
AngleA = A2 - A4

PDOF2 = 180 - (PDOF1 + A2)
AngleB = 180 - (PDOF2 + AngleA)

V1 = DV1 * Sin(AngleC) / Sin(A3)
V3 = DV1 * Sin(PDOF1) / Sin(A3)

V2 = DV2 * Sin(AngleB) / Sin(AngleA)
V4 = DV2 * Sin((180 - (AngleB + AngleA)) / Sin(AngleA)

In **Digital Momentum**, **Linear Momentum** and **360 LM**, entering an "**X**" for V3 brings up a display for entering the Longitudinal DeltaV, PDOF and Slip Angle. The other variables should be entered in their regular locations for the particular module.

The **Vectors (EDR - Momentum)** module has entry locations for all of the required variables as long as no entry is made in the Lateral DeltaV input location. It also has a **Formulae*** button that will display the interactive formulae and variables used in the computations. The vector speed solutions are displayed in Primary and Secondary output modes along with a graphical display of the collision including DeltaV/PDOF vectors.

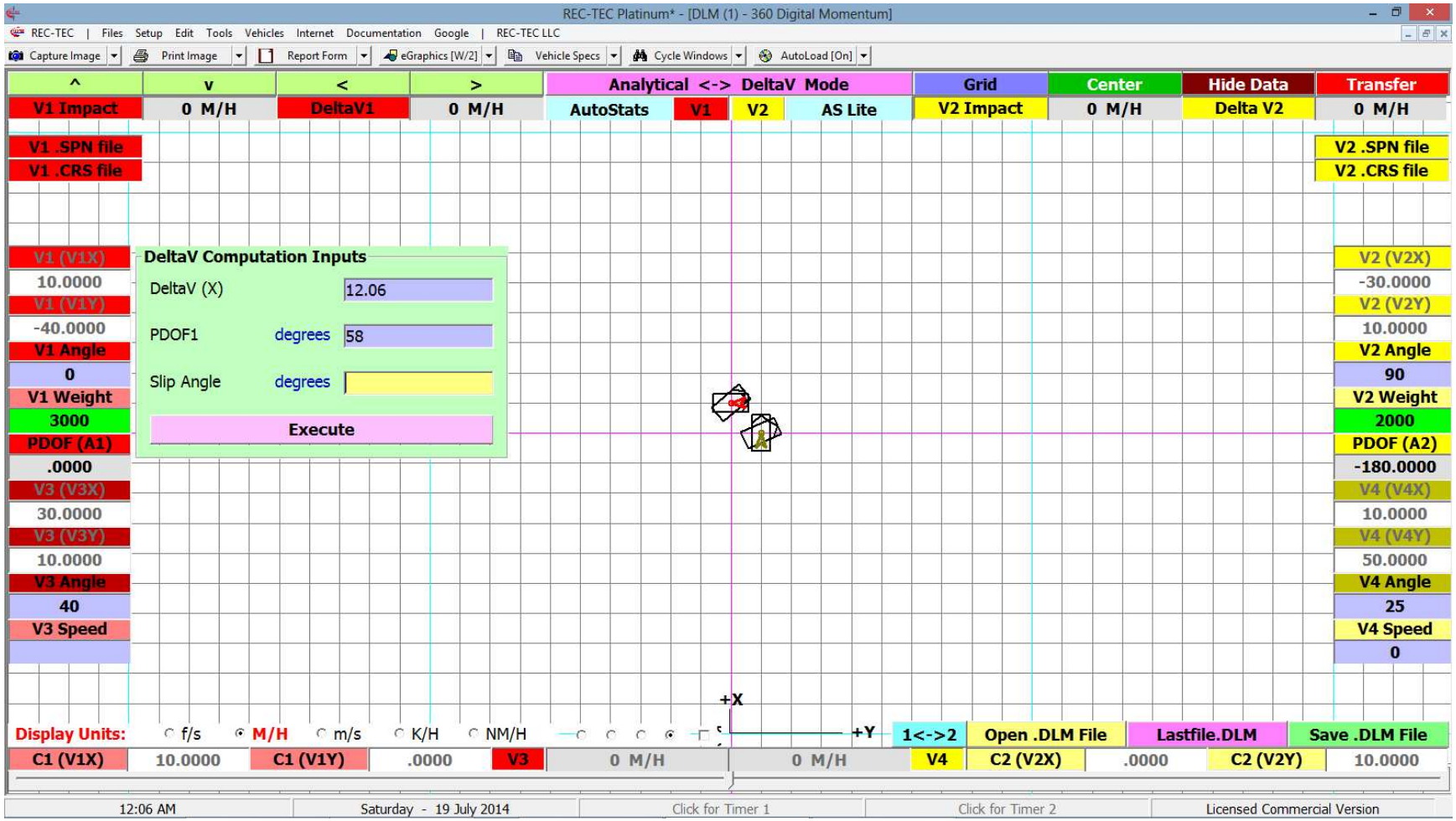
All of the Examples will use the following data except as noted:

Longitudinal DeltaV = 12.06 M/H
Angle 1 = 0 degrees
Angle 2 = 90 degrees
Angle 3 = 40 degrees
Angle 4 = 25 degrees
PDOF1 = 58 degrees
Slip Angle = 0 degrees
Weight 1 = 3000
Weight 2 = 2000

Example 1 – Digital Momentum

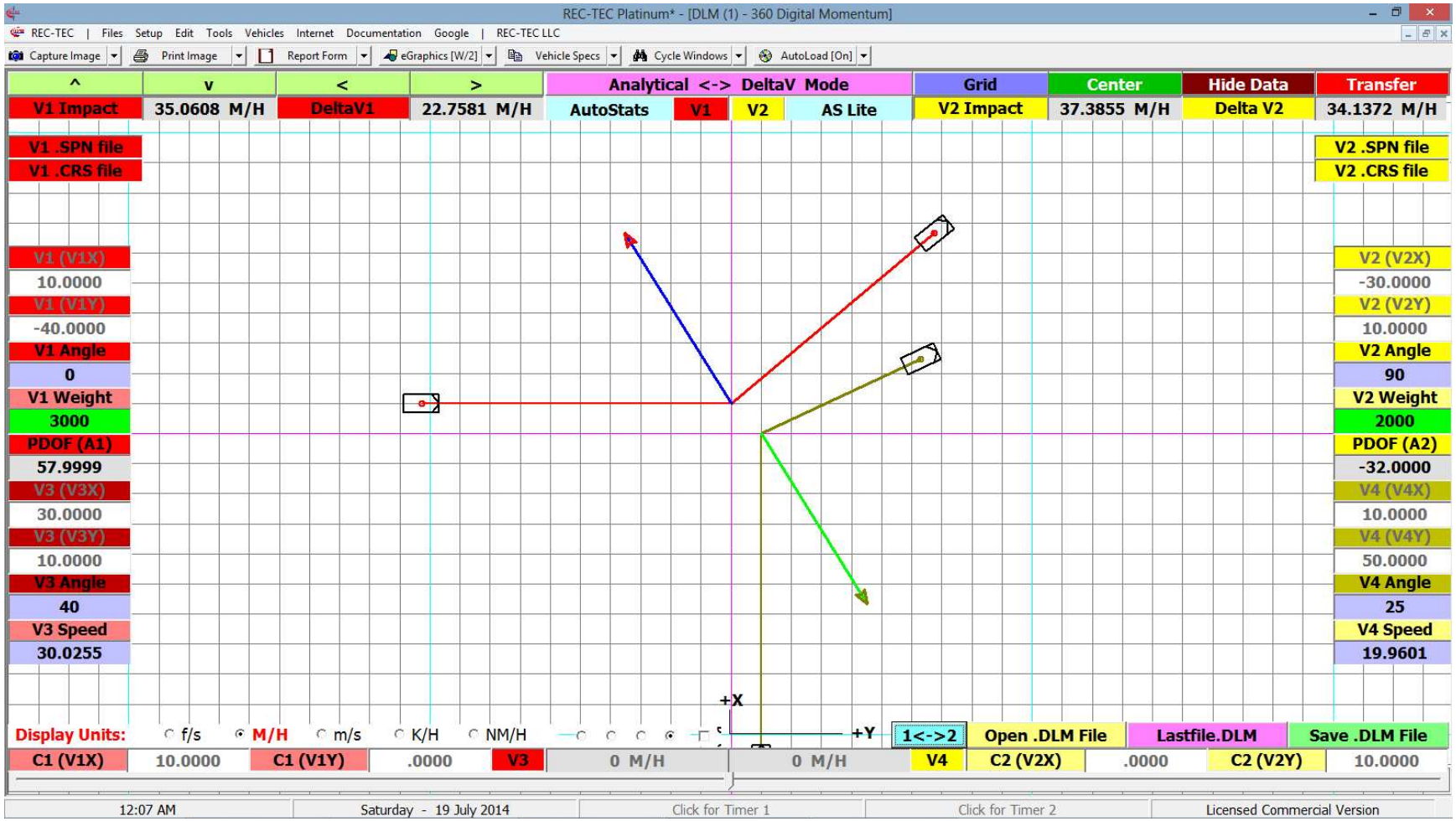
Entering an "**X**" in the V3 Speed box brings up the DeltaV Computation Inputs frame.

Figure 1: Digital Momentum



Entering the data and clicking on the **Execute** button creates Figure 2.

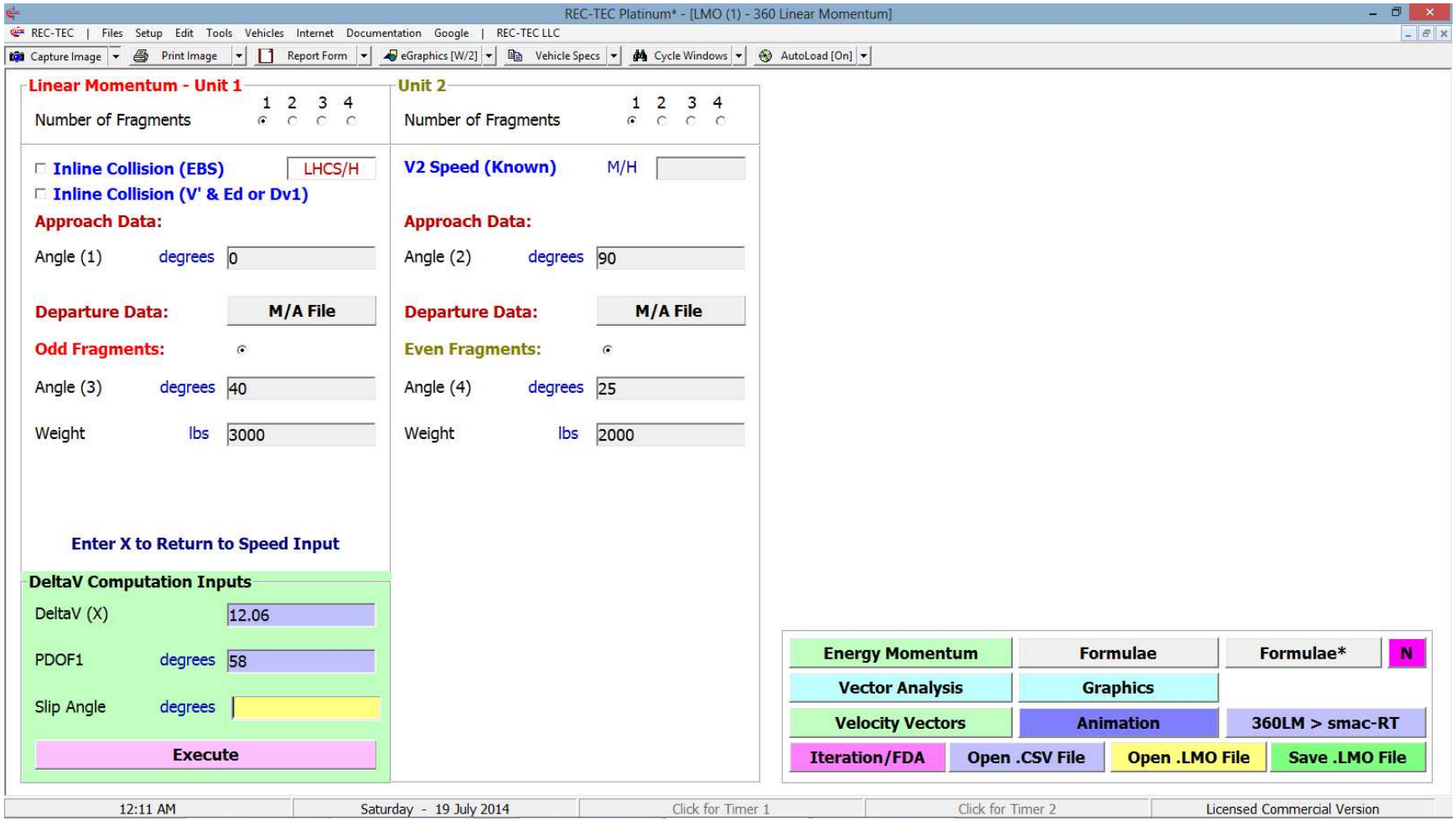
Figure 2: Digital Momentum



Example 2 – 360 Linear Momentum

Entering an "**X**" in the V3 Speed brings up the DeltaV Computation Inputs frame and hides the V3 and V4 Speed inputs.

Figure 3: 360 Linear Momentum



Entering the data and clicking on the **Execute** button creates Figure 4.

Figure 4: 360 Linear Momentum

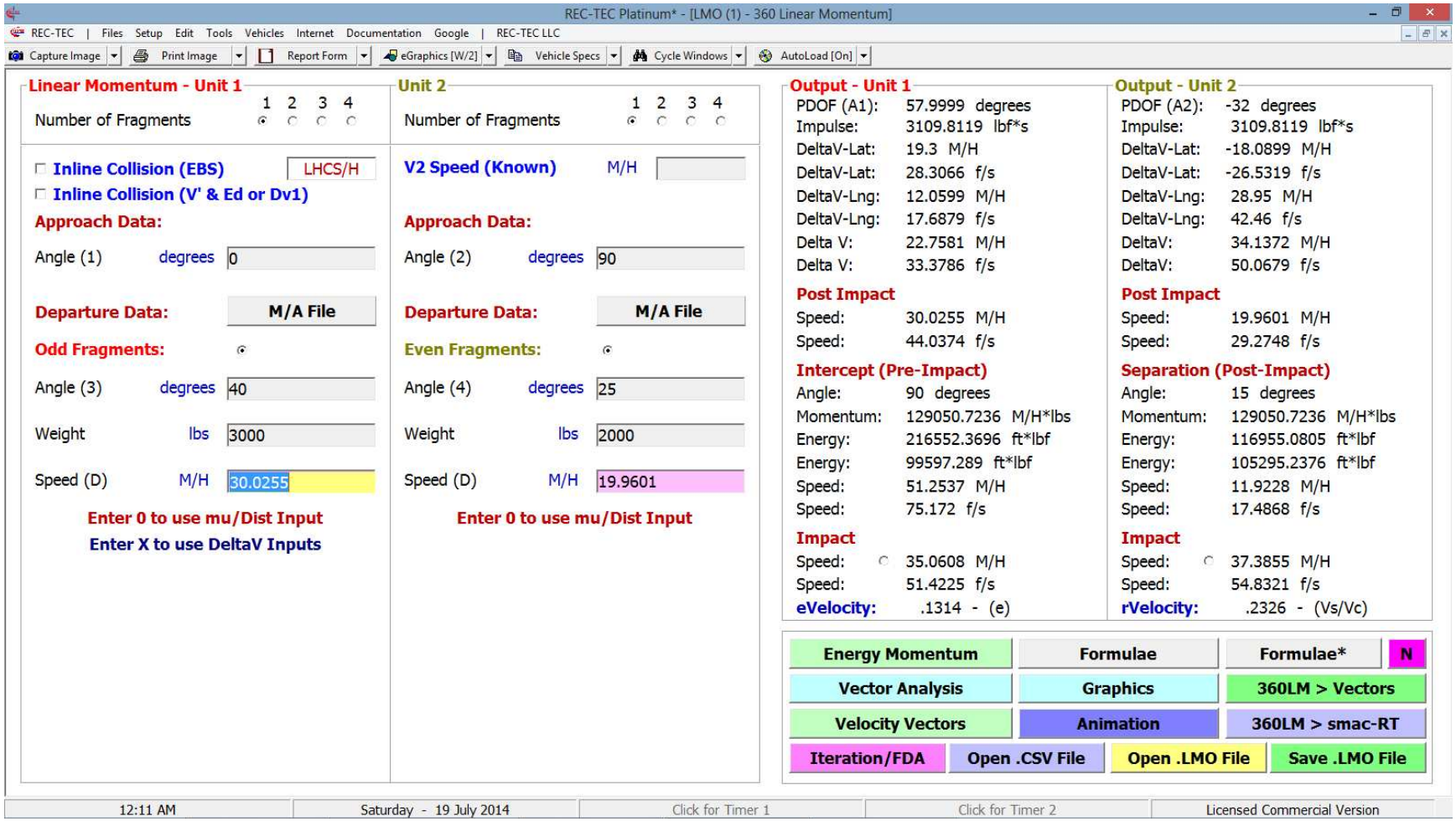
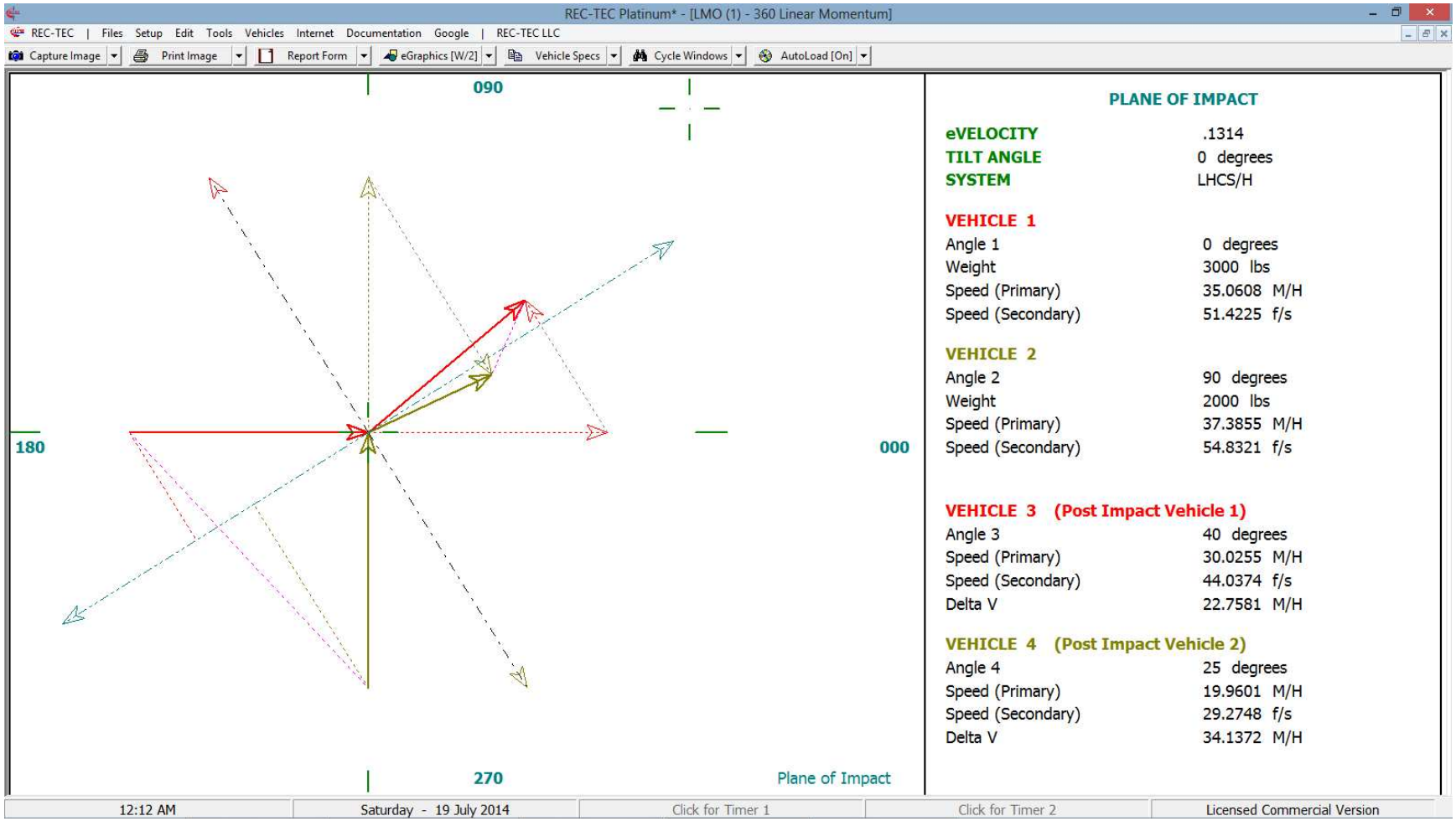


Figure 5 shows the Graphics – Right Clicking on the Graphics will bring up various Vector displays.

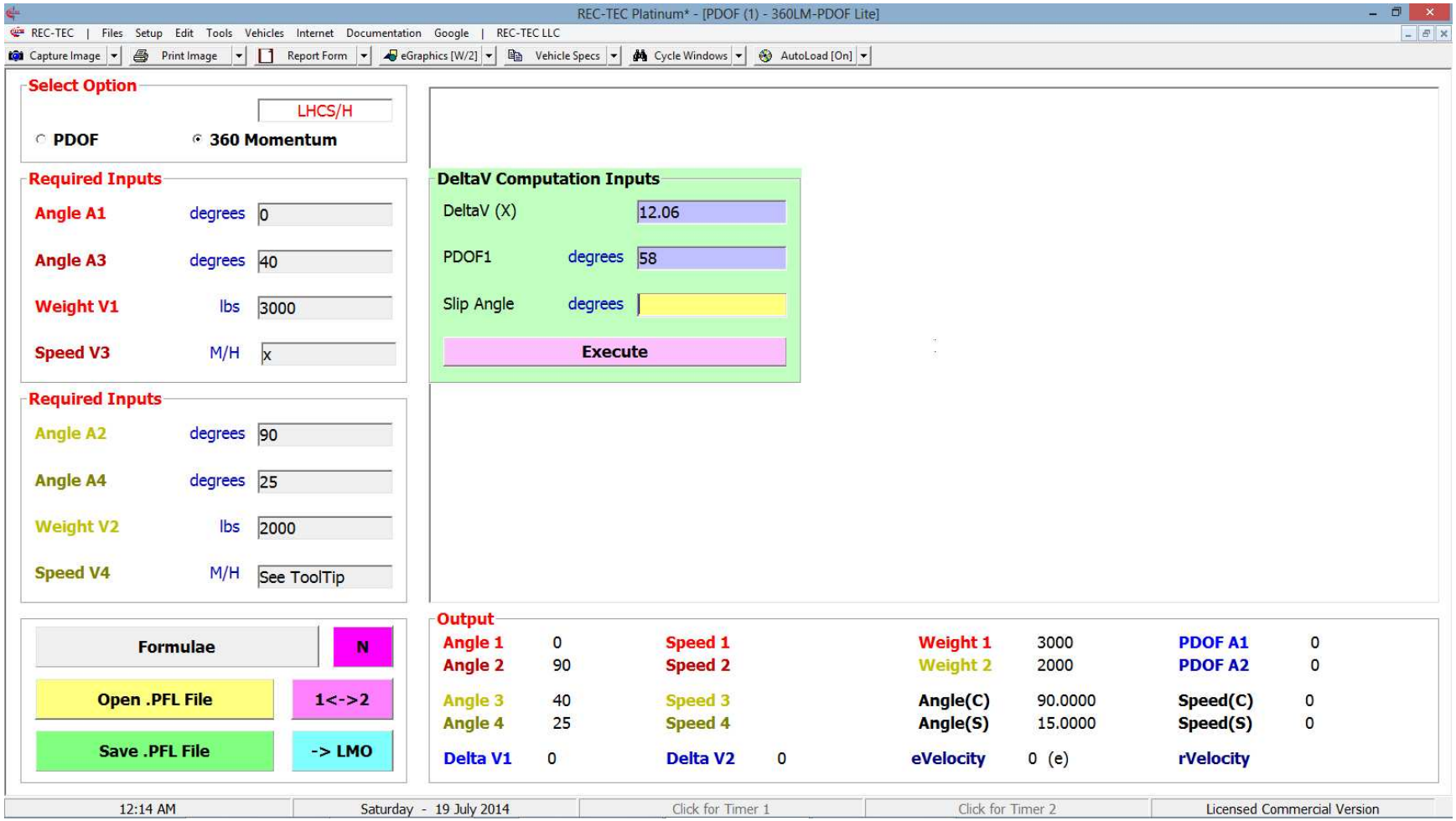
Figure 5: 360 Linear Momentum



Example 3 – 360 LM Lite

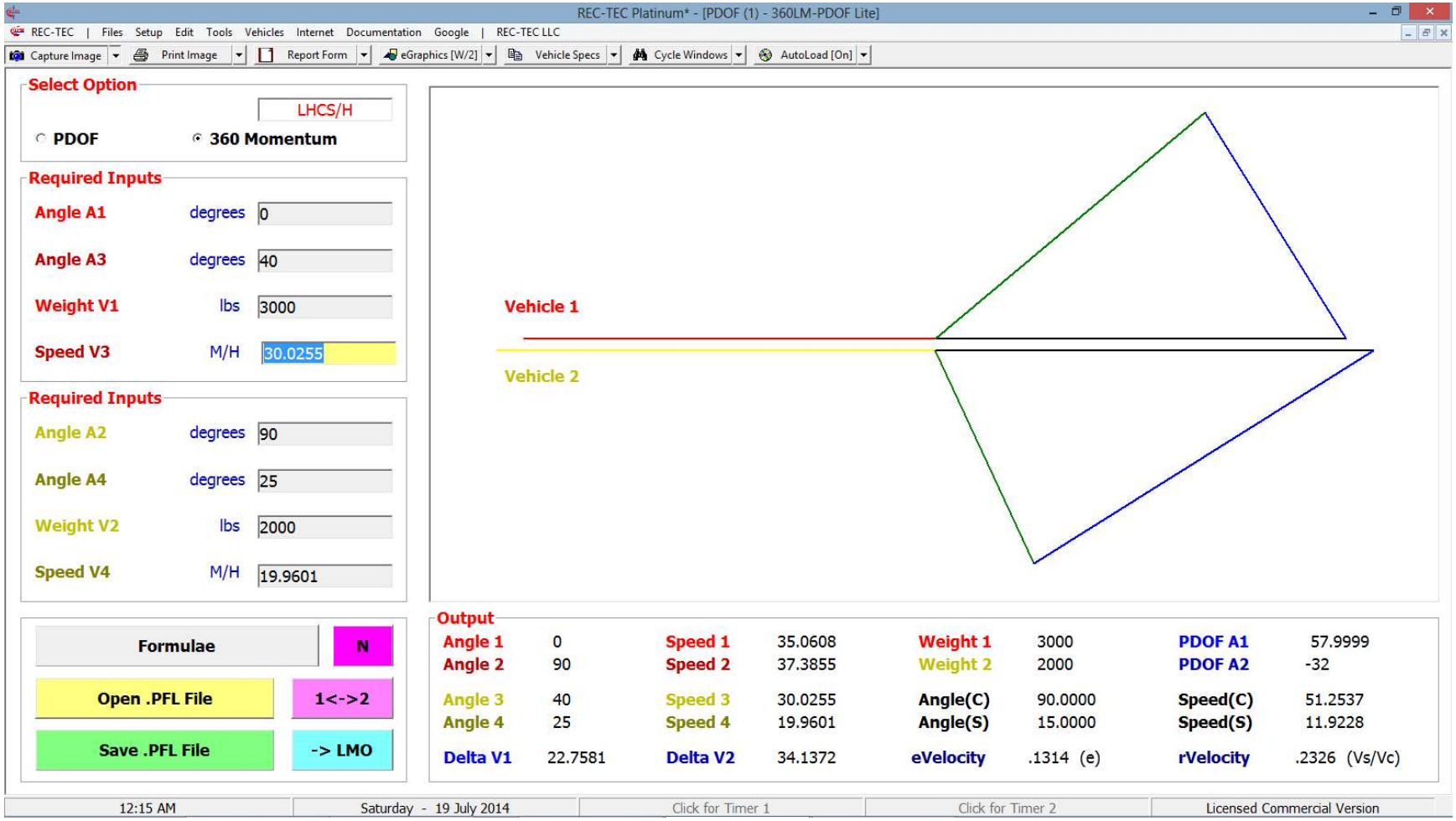
Entering an "**X**" in the V3 Speed box brings up the DeltaV Computation Inputs frame.

Figure 6: 360 LM Lite



Entering the data and clicking on the **Execute** button creates Figure 7.

Figure 7: 360 LM Lite



Example 4 – Crush

Figure 8: Crush5 - Vehicle 1

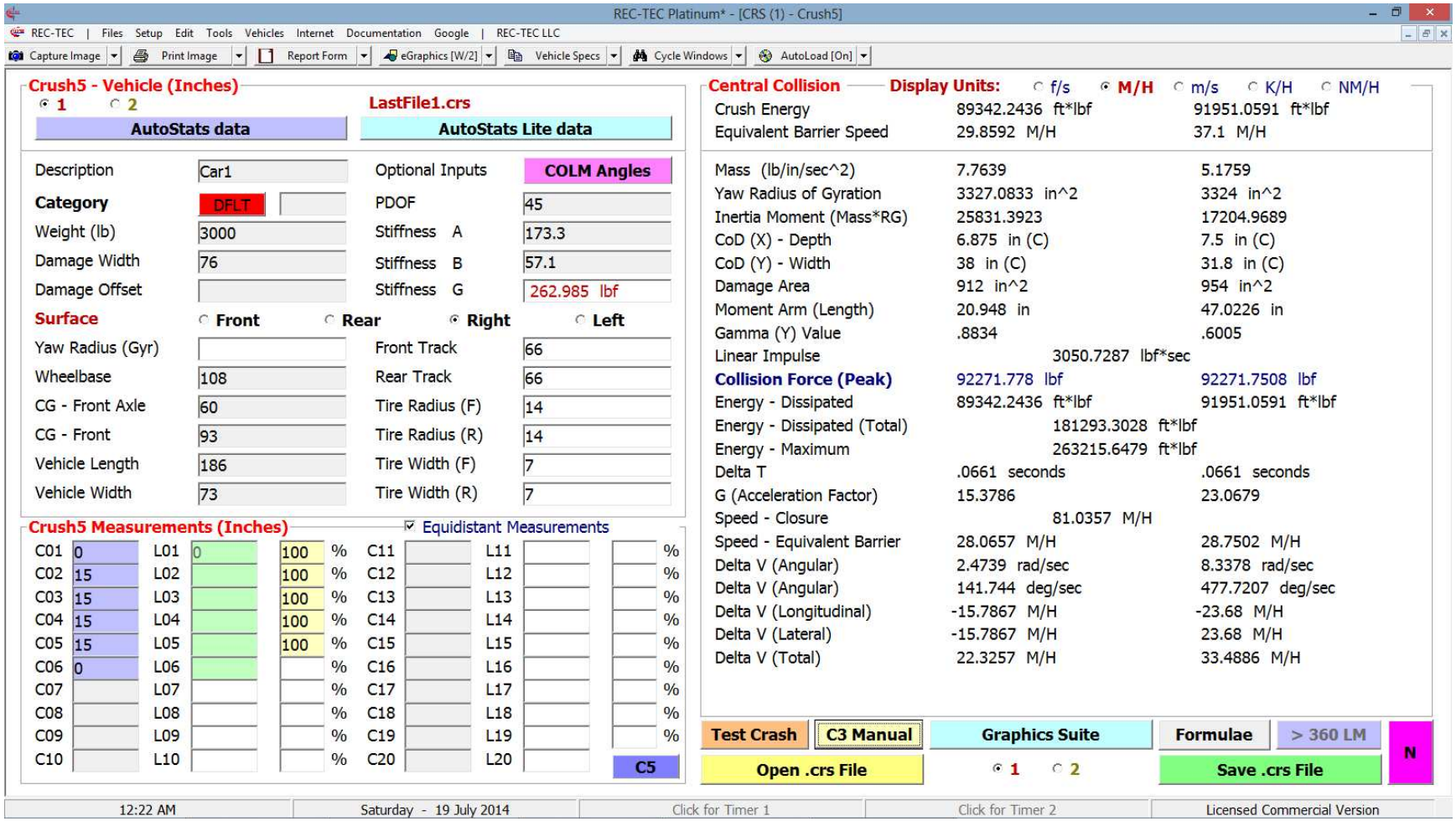
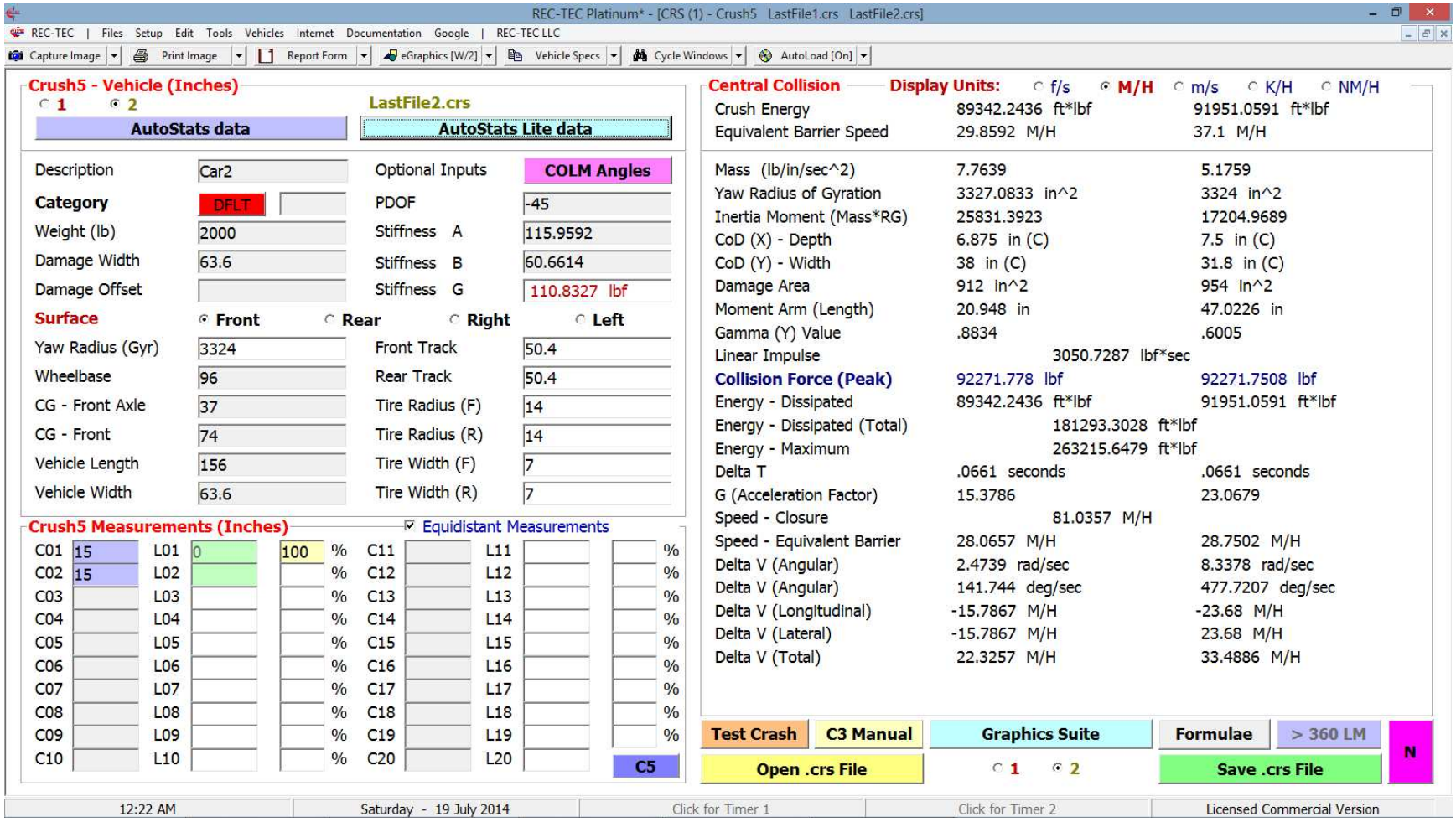


Figure 9: Crush5 - Vehicle 2



Click on **COLM** button to add angles

Figure 10: Crush5 - COLM Angles

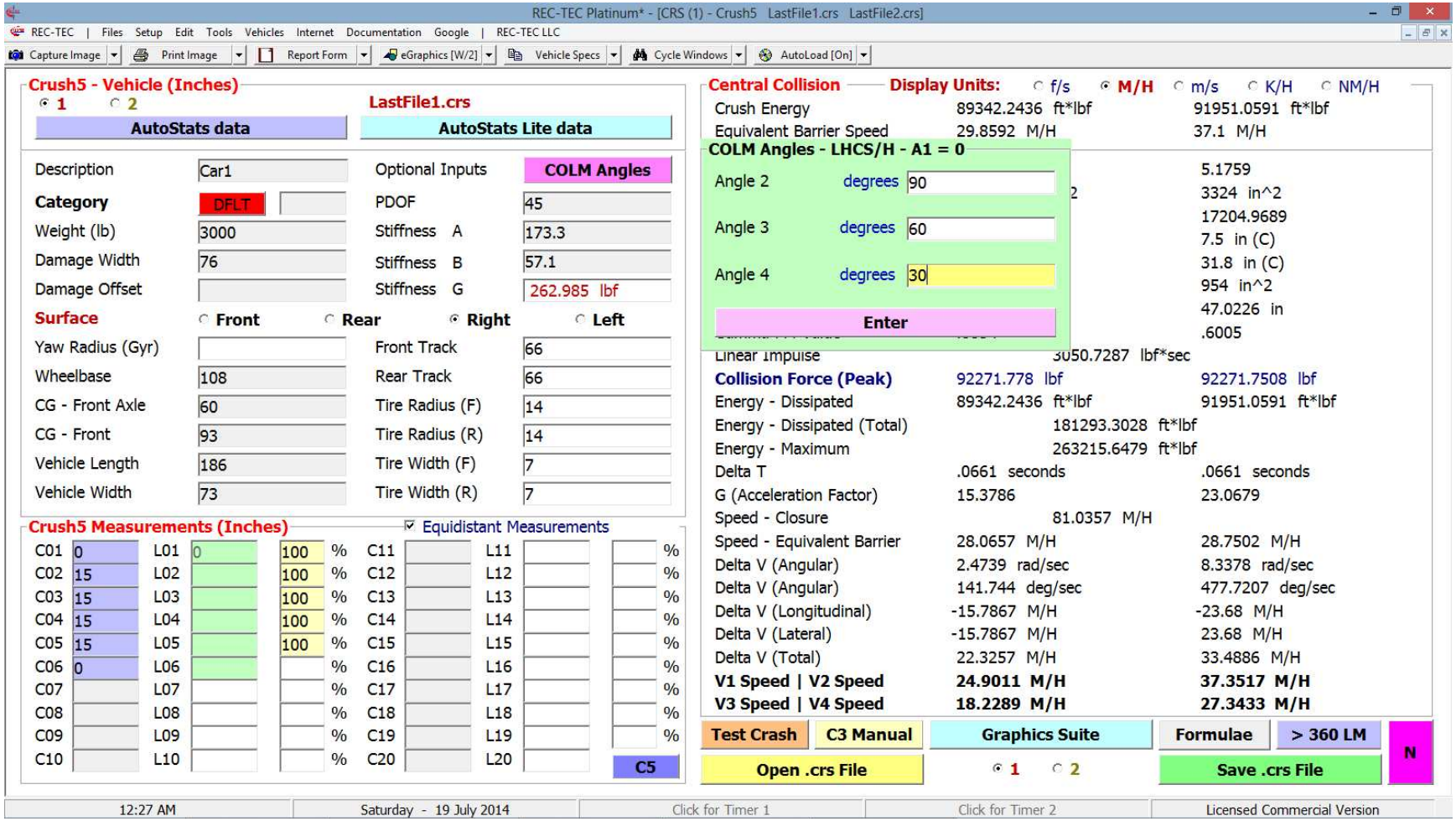
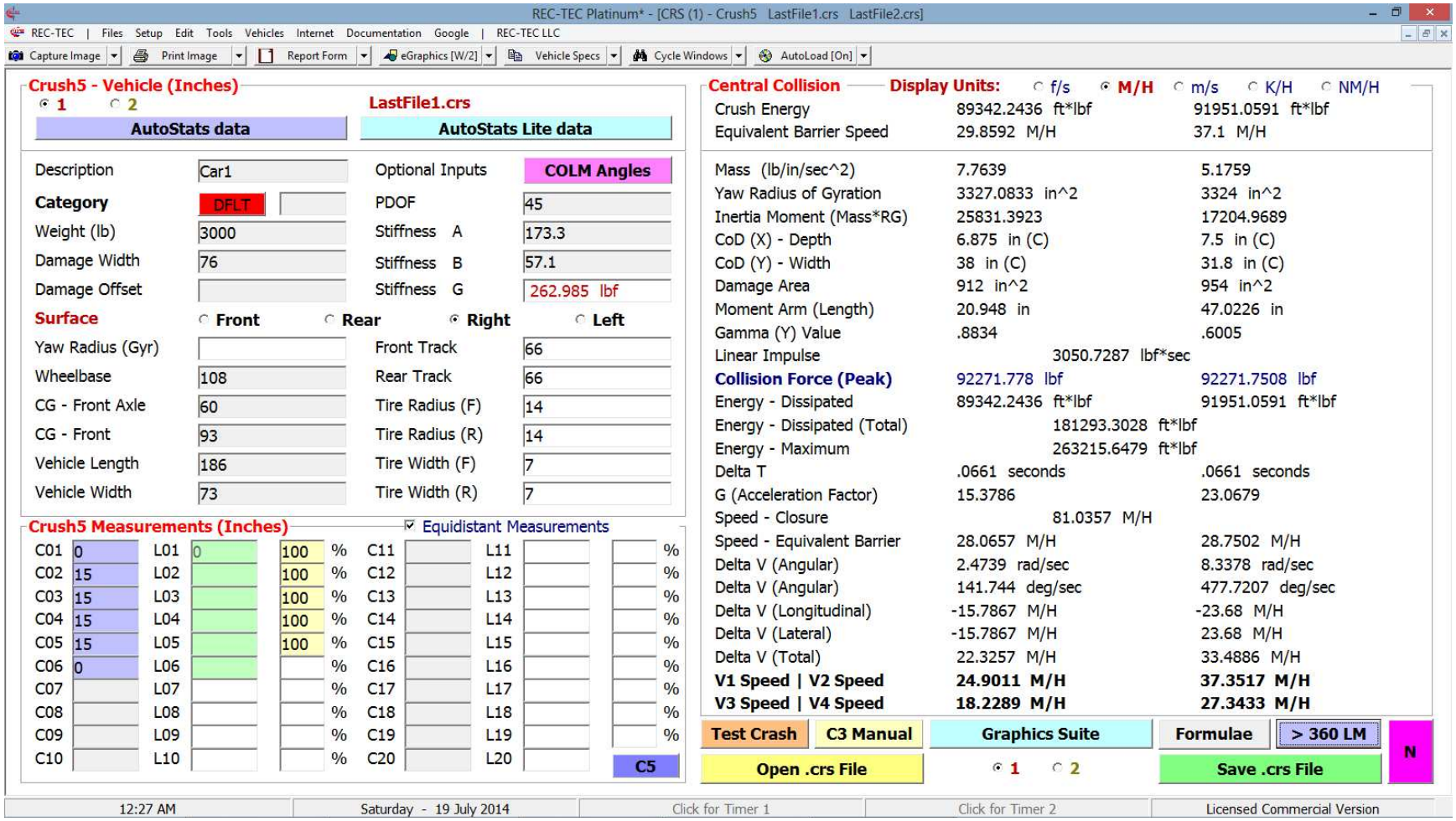
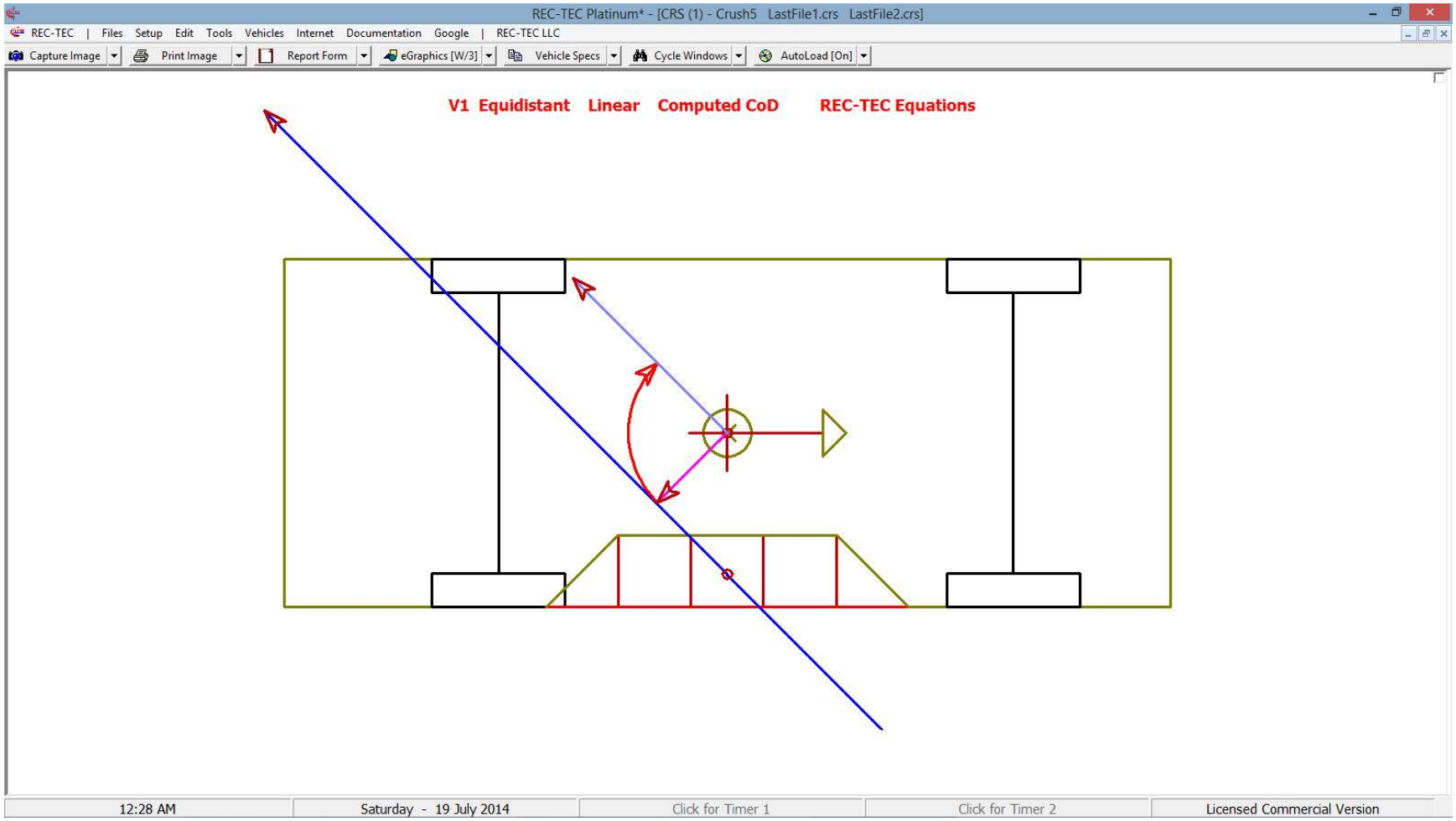


Figure 11: Crush5 - Pre Impact & Post Impact Speeds



Clicking on **Graphics Suite** button brings up Vehicle 1 Graphics

Figure 12: Crush5 - Vehicle 1 Graphics



Use the **Escape [Esc]** key to advance to next Screen or **Right** click on Graphics to Re-draw the graphics.

Figure 13: Crush5 - Vehicle 2 Graphics

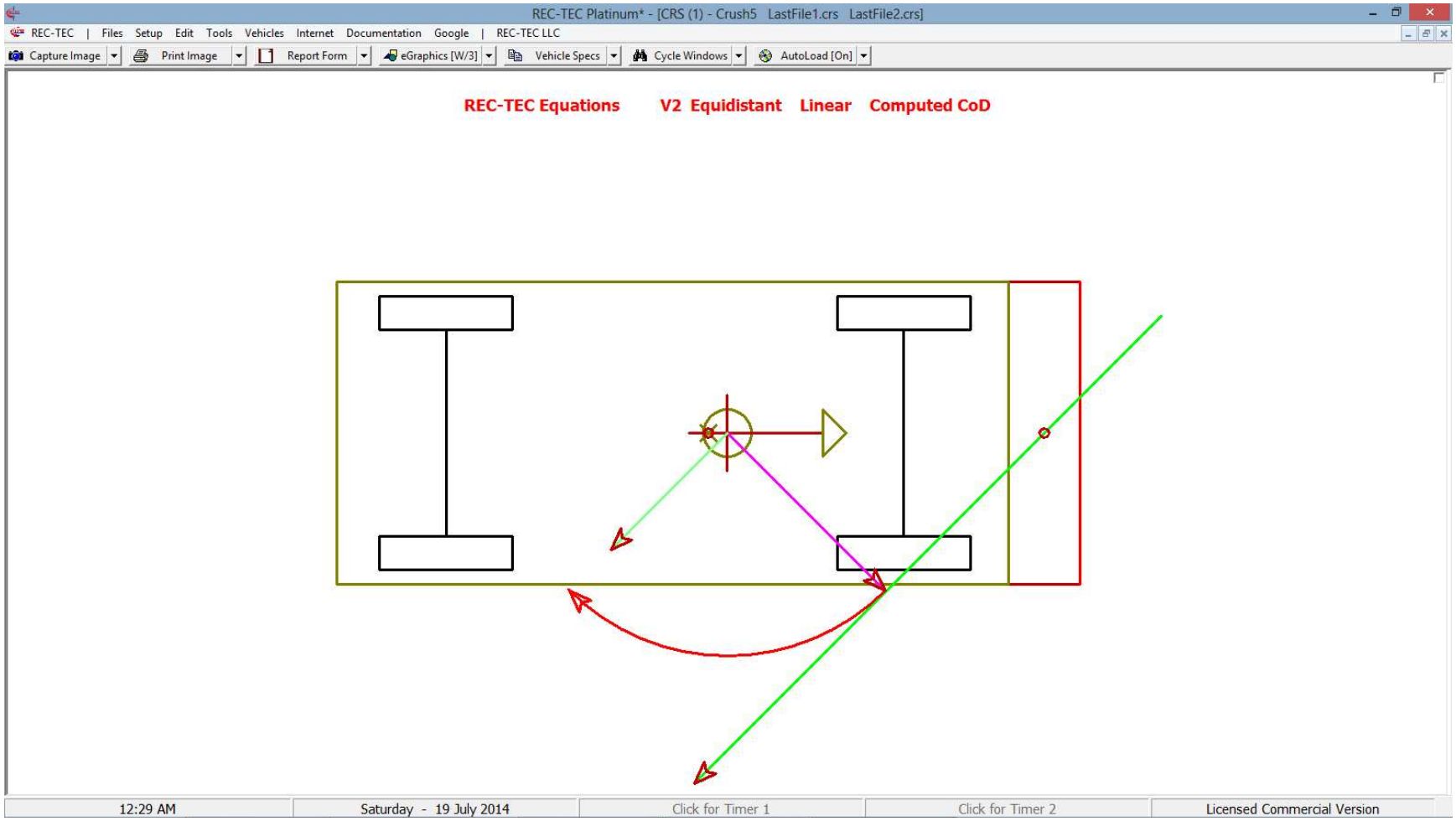


Figure 14: Crush5 - Vehicles 1 & 2 (Side-by-Side)

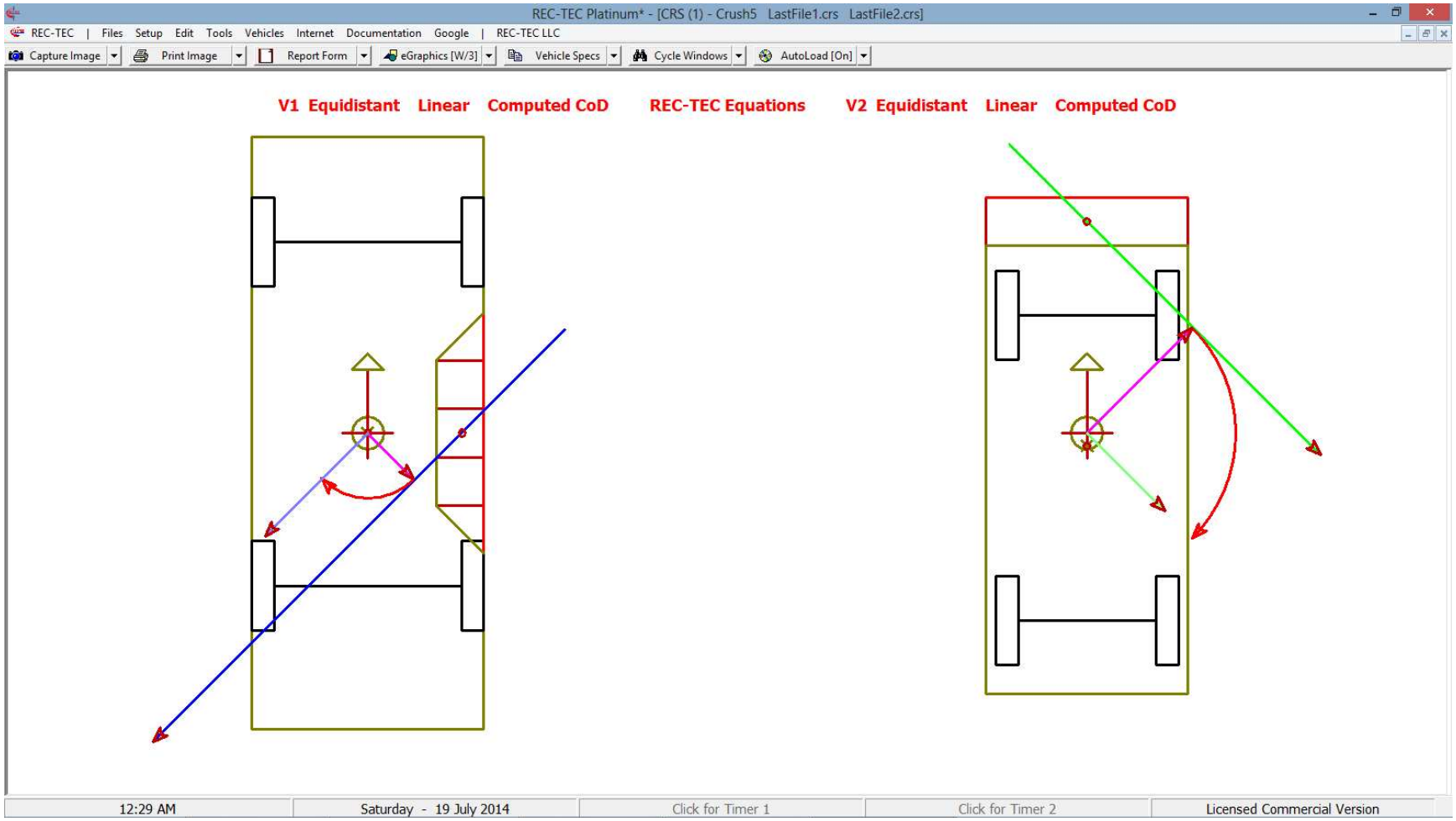


Figure 15: Crush5 - Vehicles 1 & 2 (Maximum Engagement)

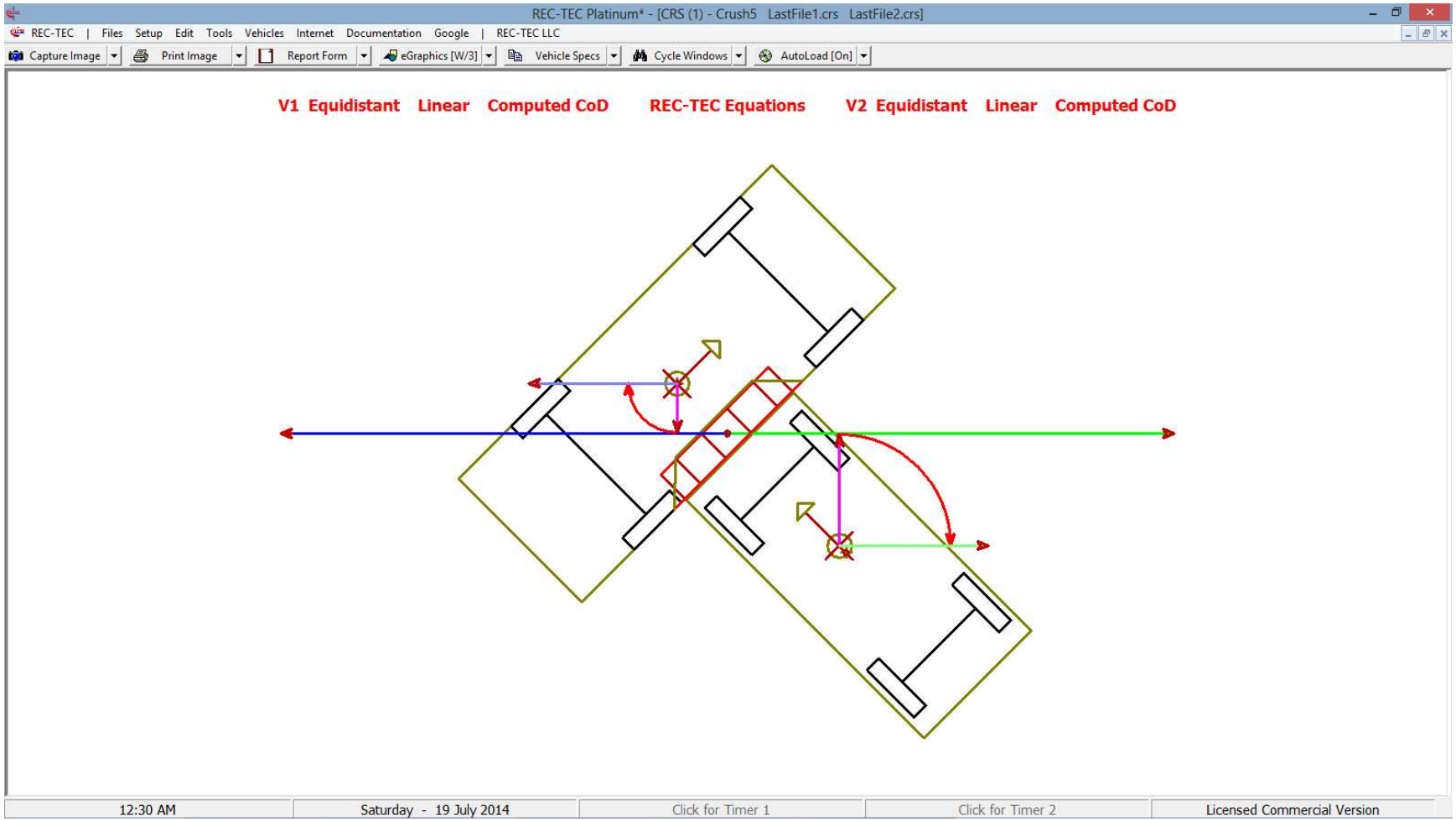
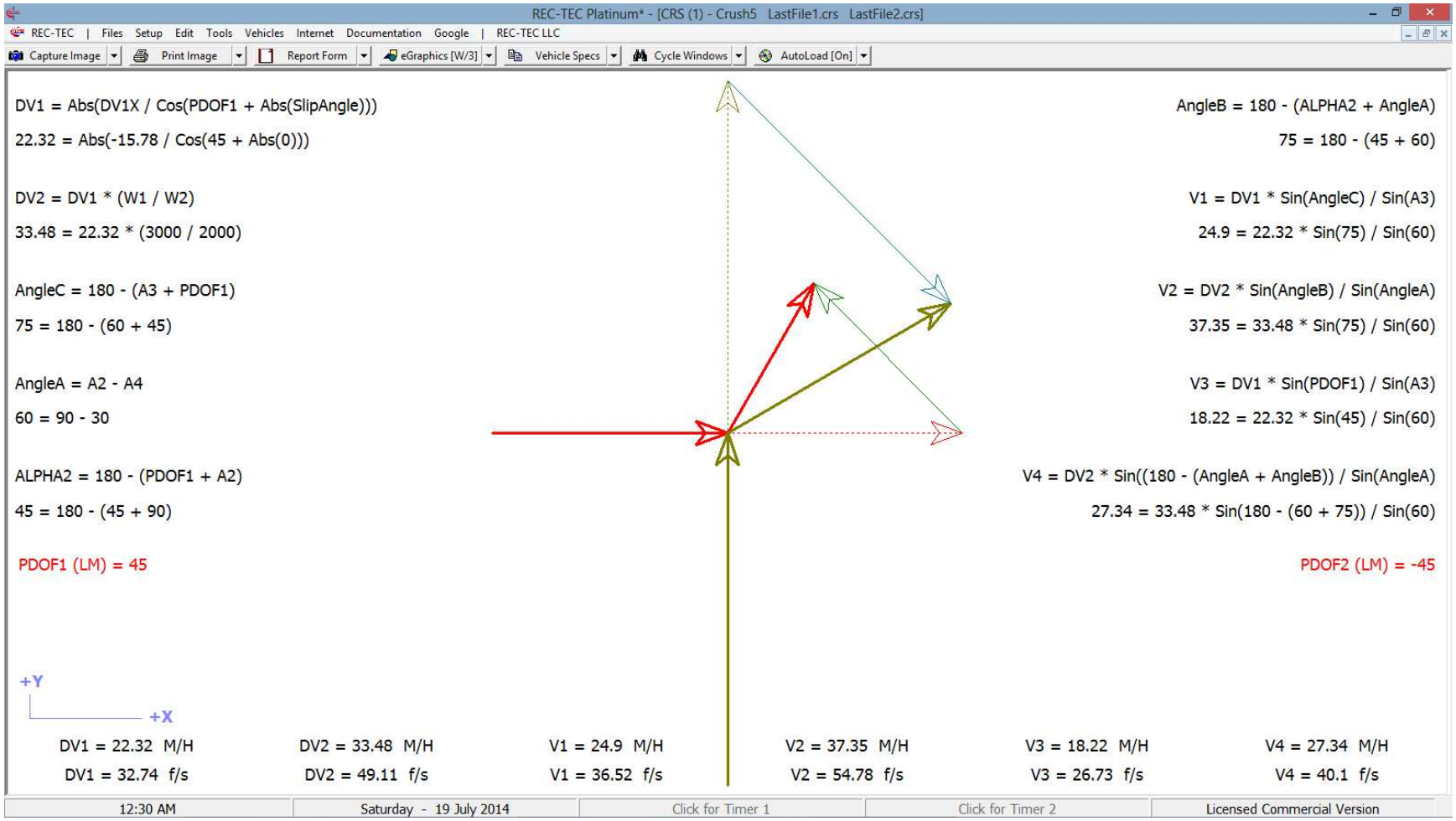


Figure 16: Crush5 - Vehicles 1 & 2 (Vector Solution)

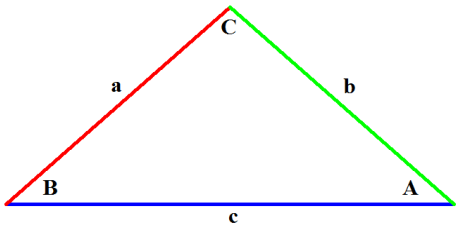


Triangulation:

Before diving into **Vectors (EDR-Momentum)** a little history on how and why this works is in order. We will be working with Triangles. There are a lot of AI/AR problems that can be solve using Triangles. In using Triangles to solve problems, we must first know how to solve for the missing parts of Triangles.

Some basics about Triangles:

- All Triangles have exactly 3 sides - usually lower case letters (a, b, and c)
- All Triangles have 3 interior angles - usually upper case letters (A, B, and C) which are opposite from the side with the same (lower case) letter
- The sum of the interior angles (between the sides) of the triangle of the Triangle equals 180 degrees
- Right Triangles have one of the interior angles equal to 90 degrees
    - $c^2 = a^2 + b^2$ or $c = Sqr(a^2 + b^2)$ is called the Pythagorean Theorem it is used to solve for the missing side of a Right Triangle
- The Law of Sines and the Law of Cosines are used to solve for the missing parts of any triangle



    - Law of Sines: **SinA/a = SinB/b = SinC/c**
        - **a = b * Sin(A) / Sin(B)**
        - **b = c * Sin(B) / Sin(C)**
        - **c = a * Sin(C) / Sin(A)**
    - Law of Cosines:
        - **$a^2 = b^2 + c^2 - 2bc * Cos(A)$**
        - **$b^2 = a^2 + c^2 - 2ac * Cos(B)$**
        - **$c^2 = a^2 + b^2 - 2ab * Cos(C)$**

Figure 17 is a screen capture from REC-TEC with labels identifying the components of the triangles shown in the graphics.

Figure 17: Vectors (EDR-Momentum) with Labels

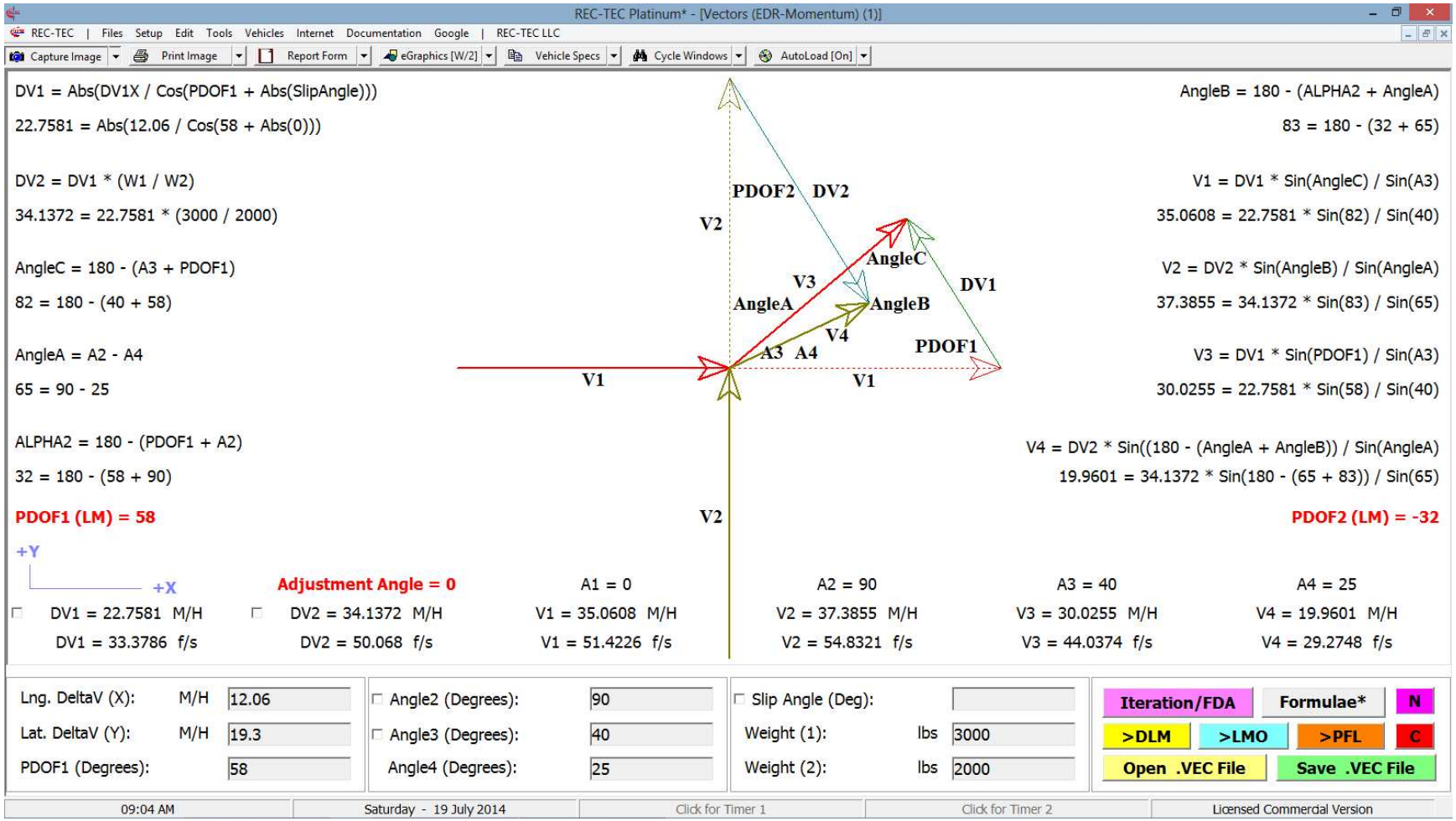
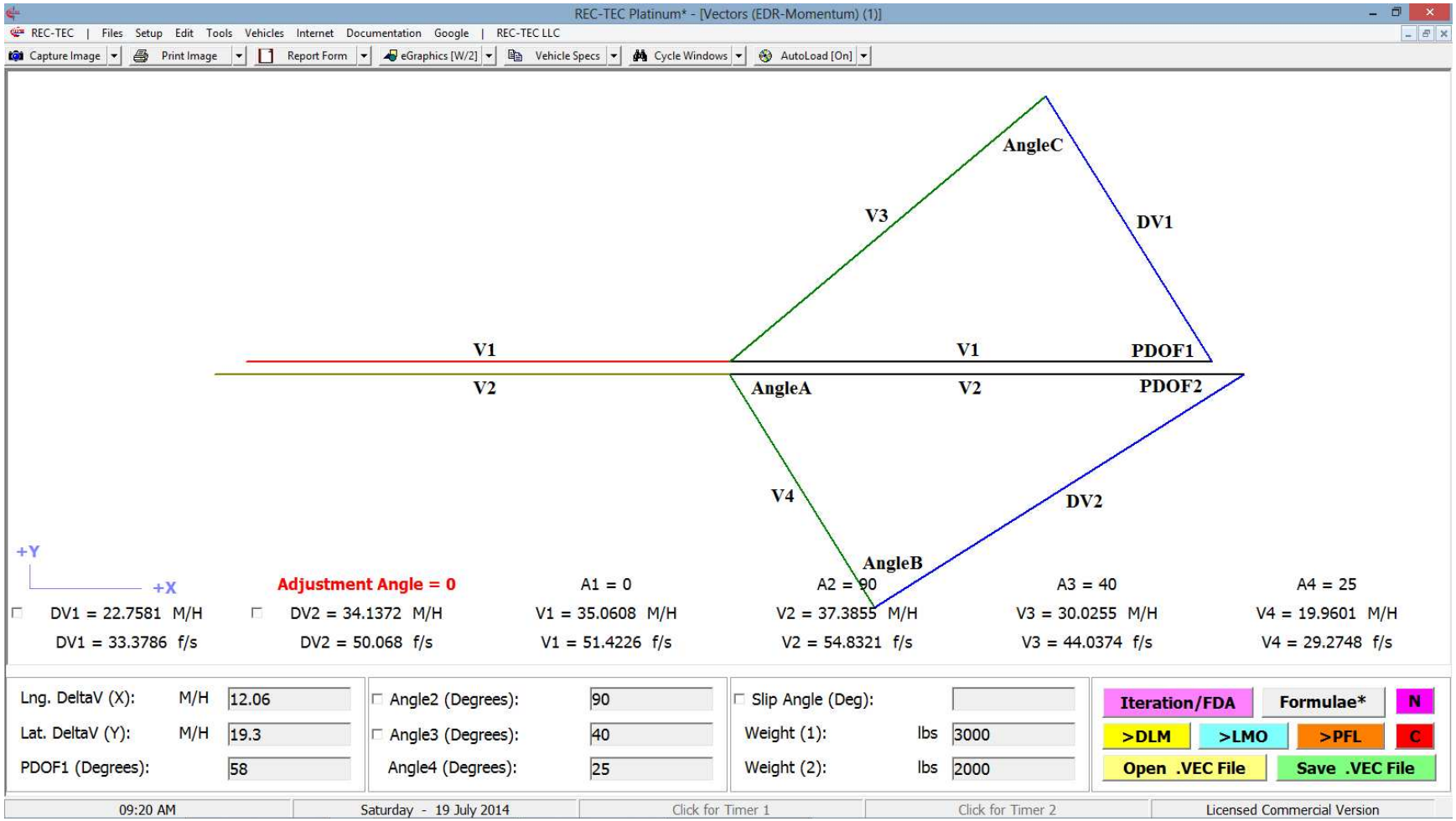


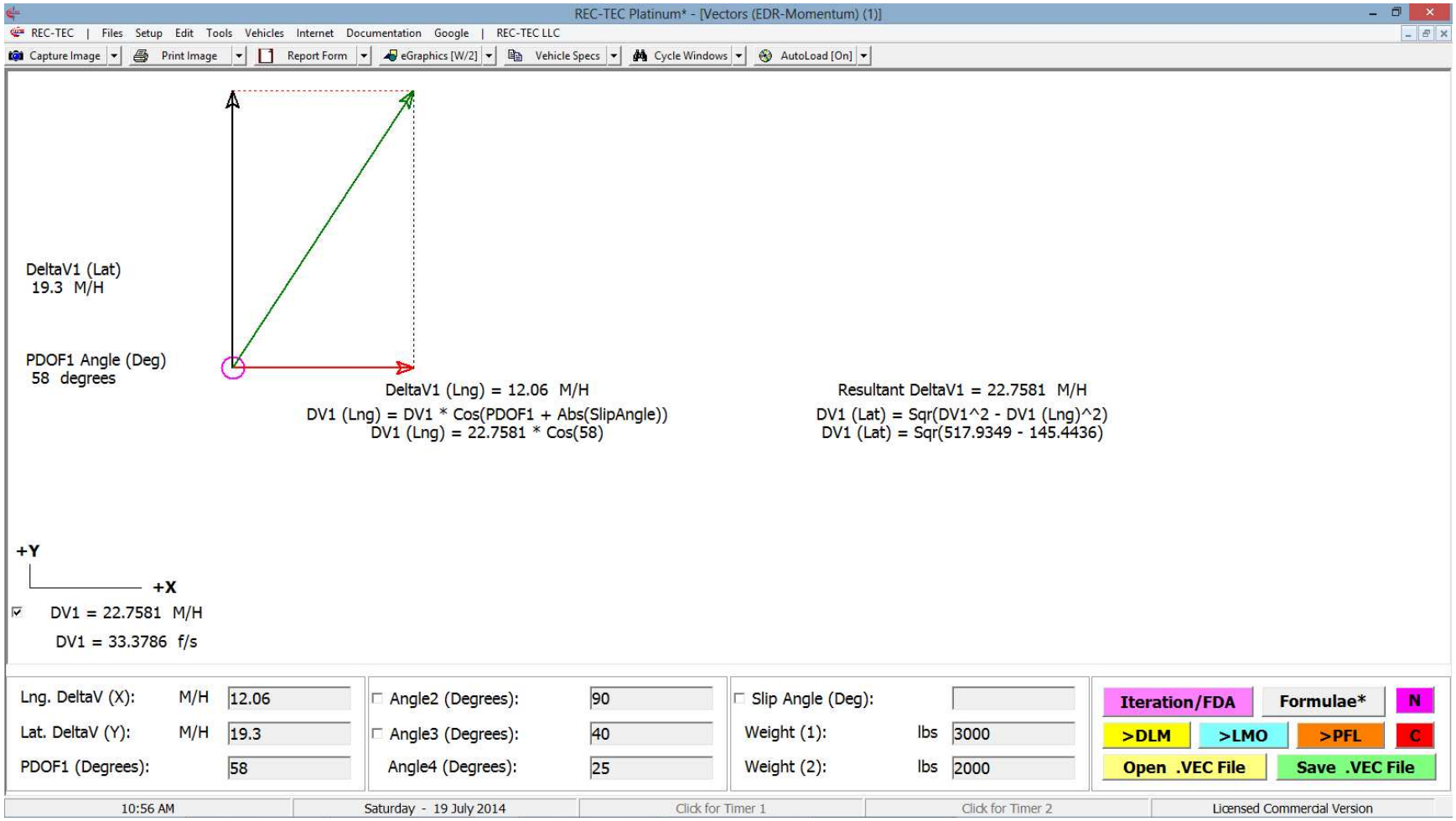
Figure 18 is a screen capture from REC-TEC with labels identifying the components of the triangles shown in the graphics.

Figure 18: Vectors (EDR-Momentum) with Labels



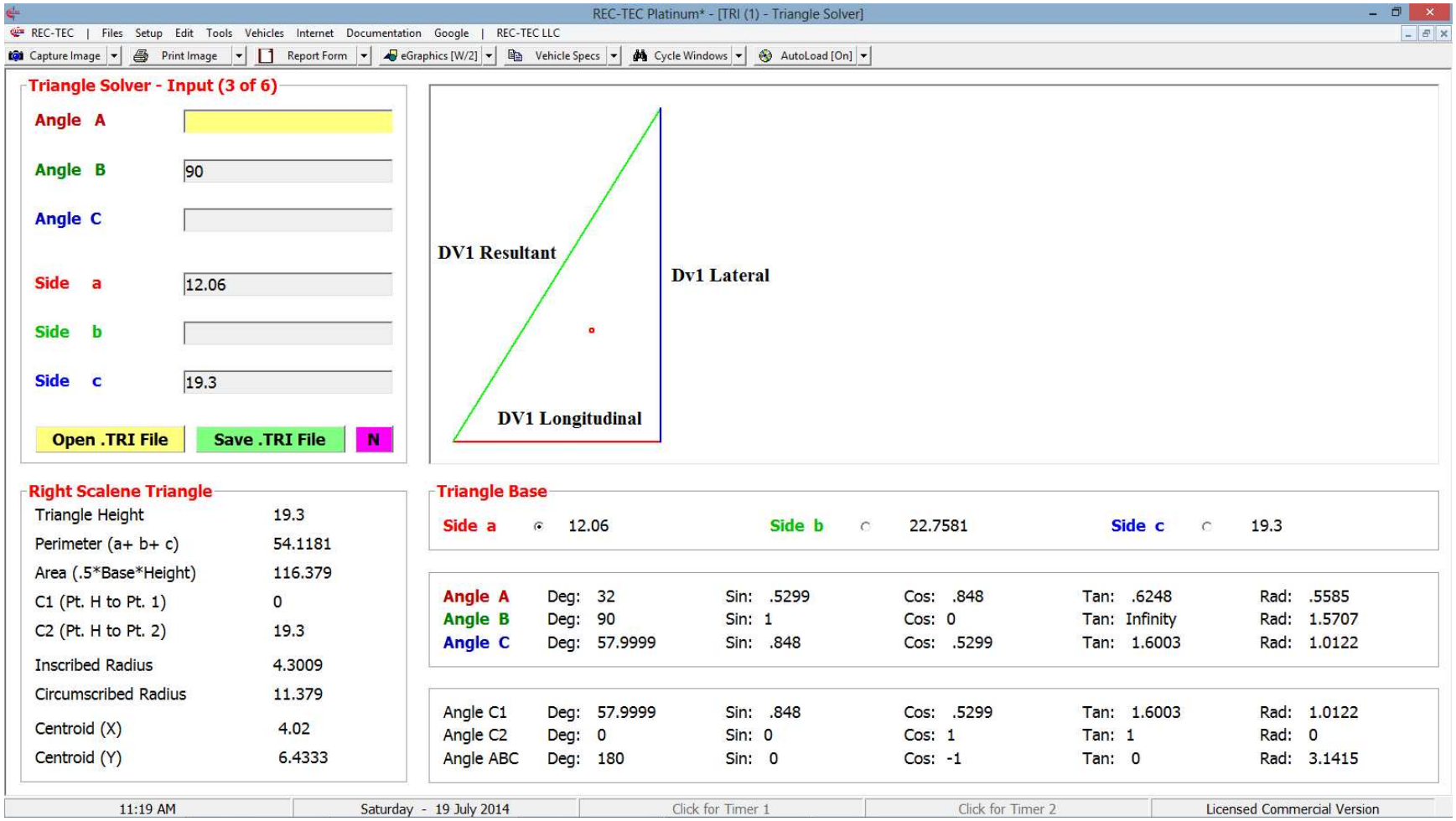
Solutions:

Figure 19: Vectors (EDR-Momentum) - DeltaV1 computation



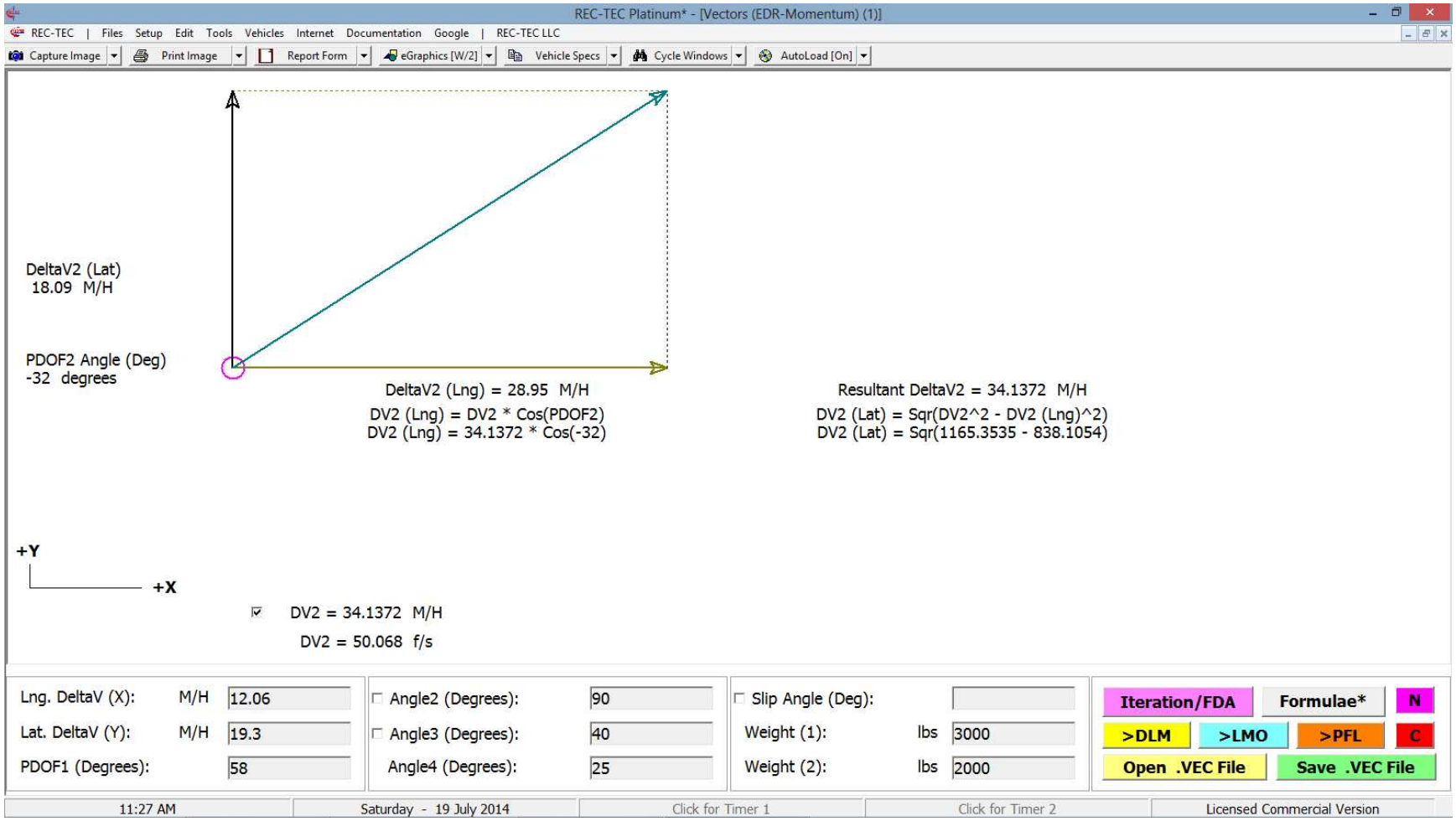
Vectors (EDR-Momentum) Solution for DV1

Figure 20: Triangle Solver - DeltaV1 Solution



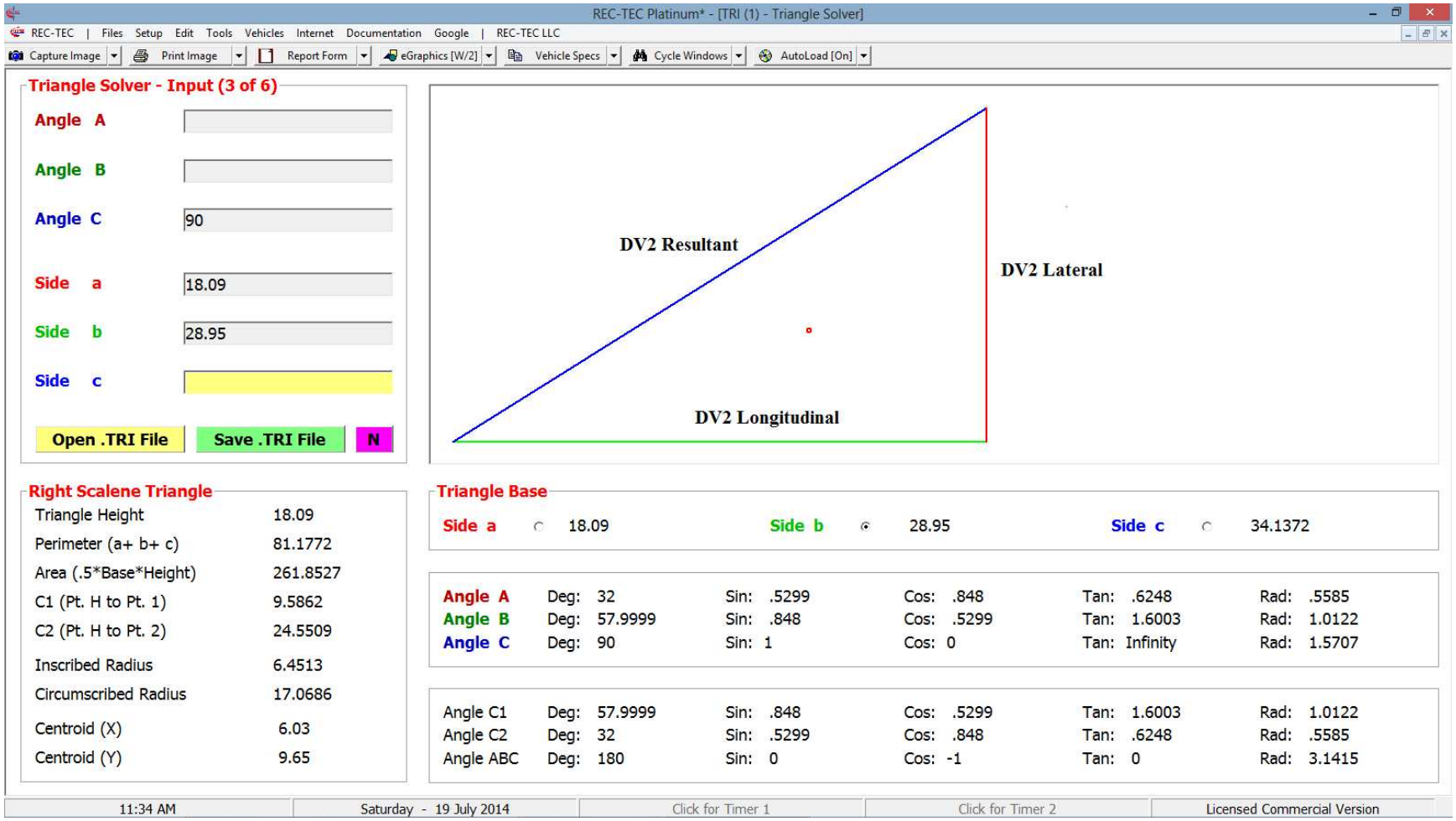
Triangle Solution for DV1

Figure 21: Vectors (EDR-Momentum) - DeltaV2 computation



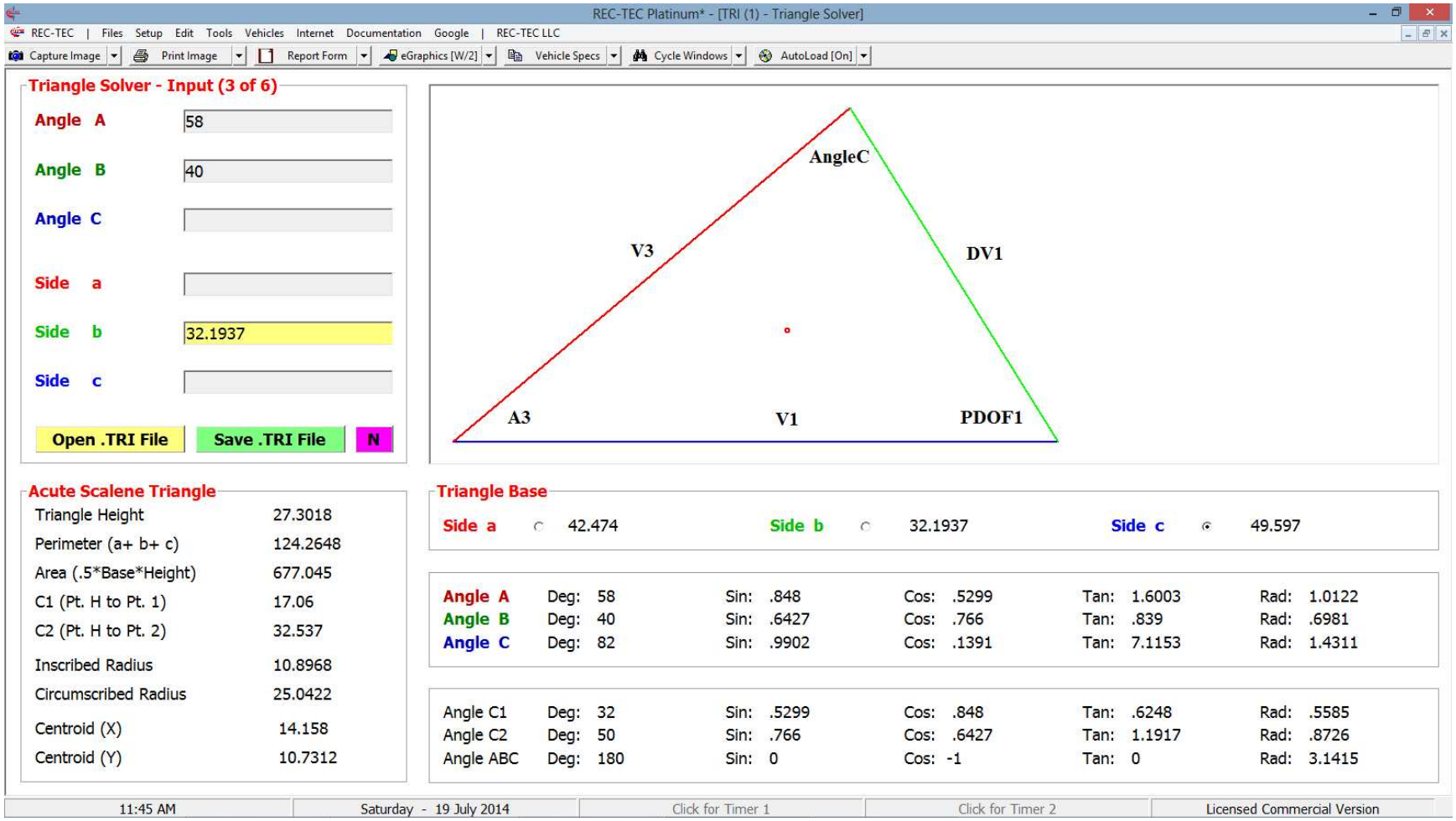
Vectors (EDR-Momentum) Solution for DV2

Figure 22: Triangle Solver - DeltaV2 Solution



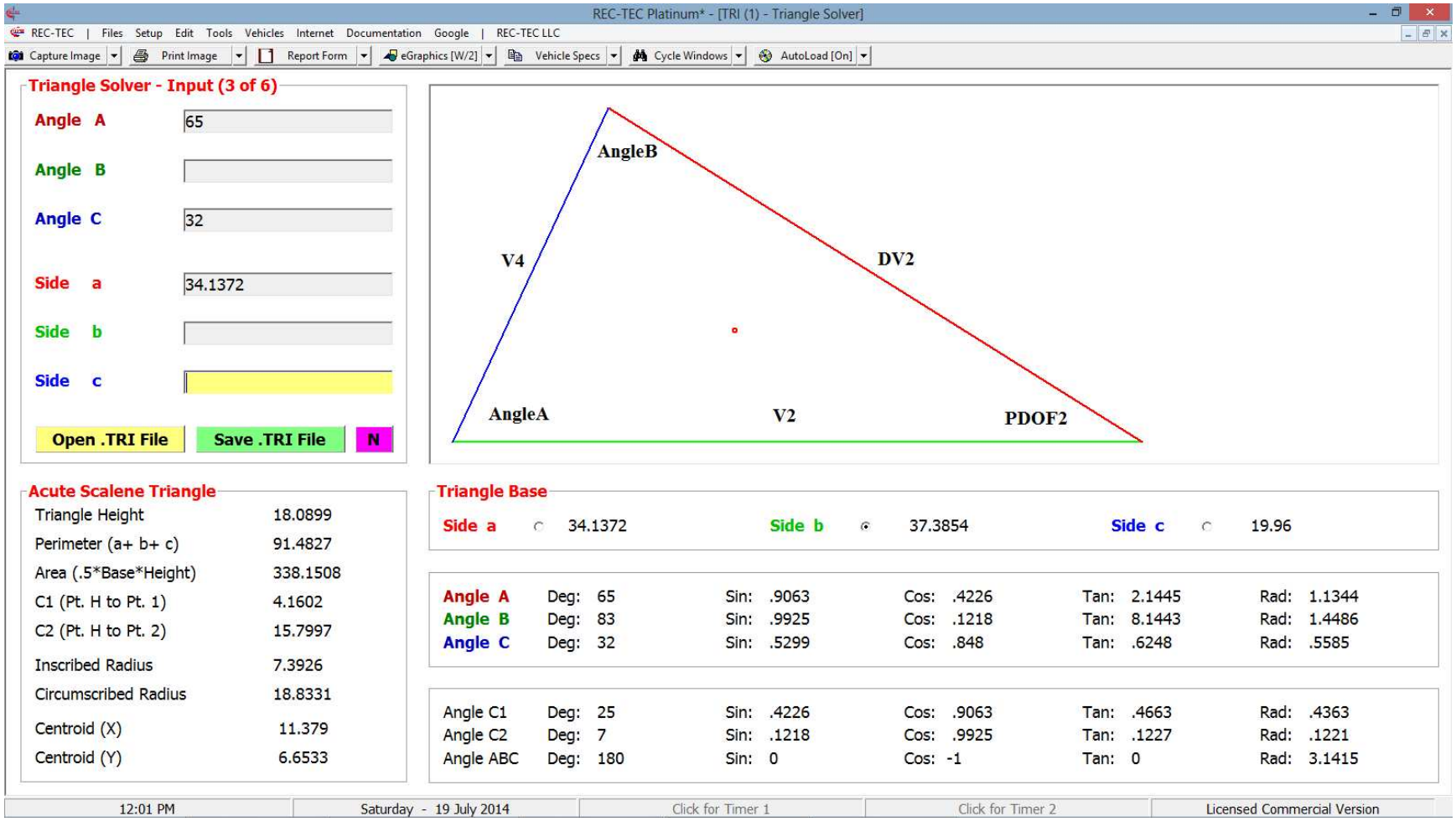
Triangle Solution for DV2

Figure 23: Triangle Solver - Solution for V1, V3, and AngleC



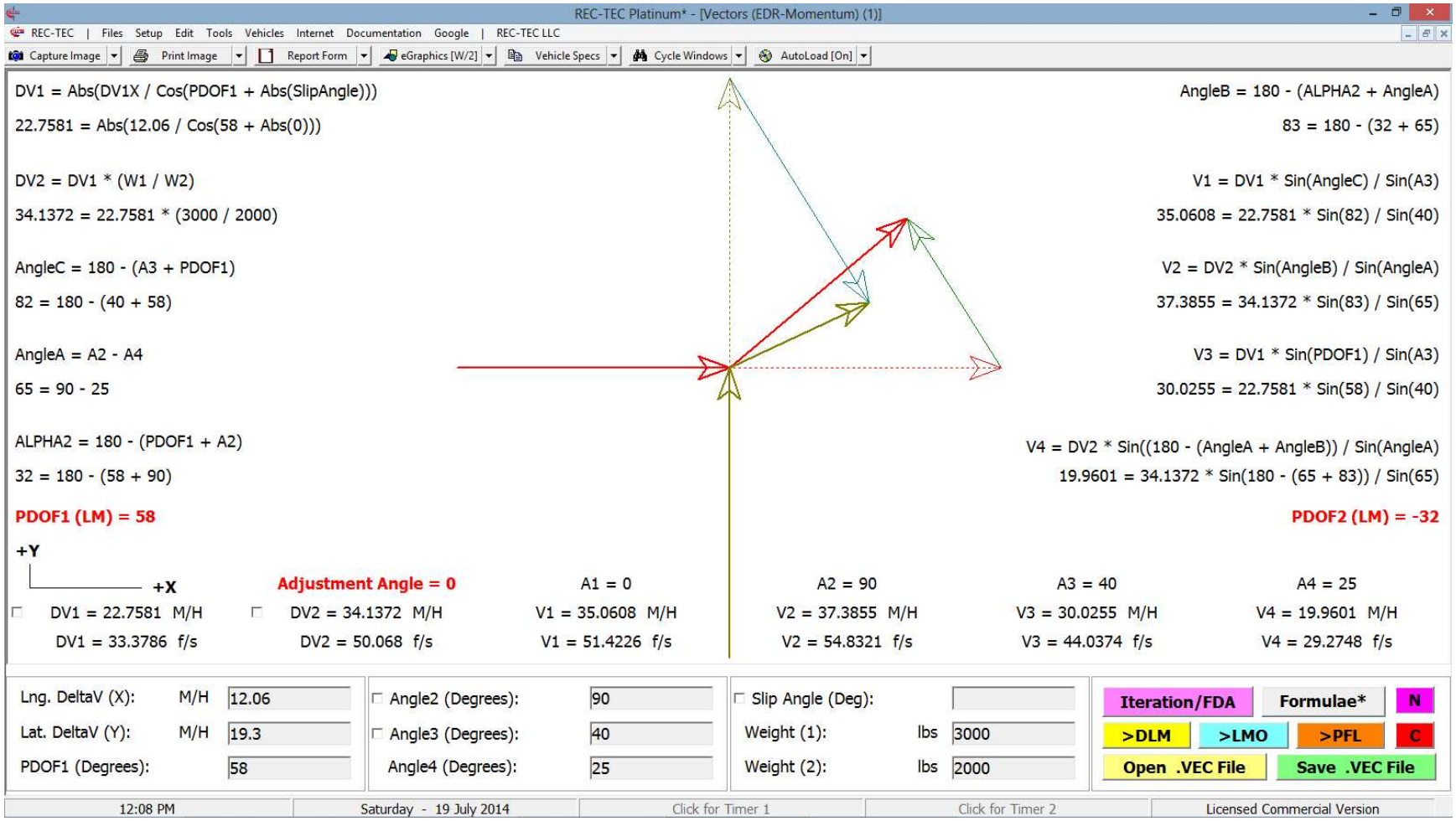
Triangle Solution for V1, V3, and AngleC
Known: PDOF1 = 58, A3 = 40, and DV1 = 32.1937
Solution: V1 = 49.597, V3 = 42.474, and AngleC = 82

Figure 24: Triangle Solver - Solution for V2, V4, and AngleB



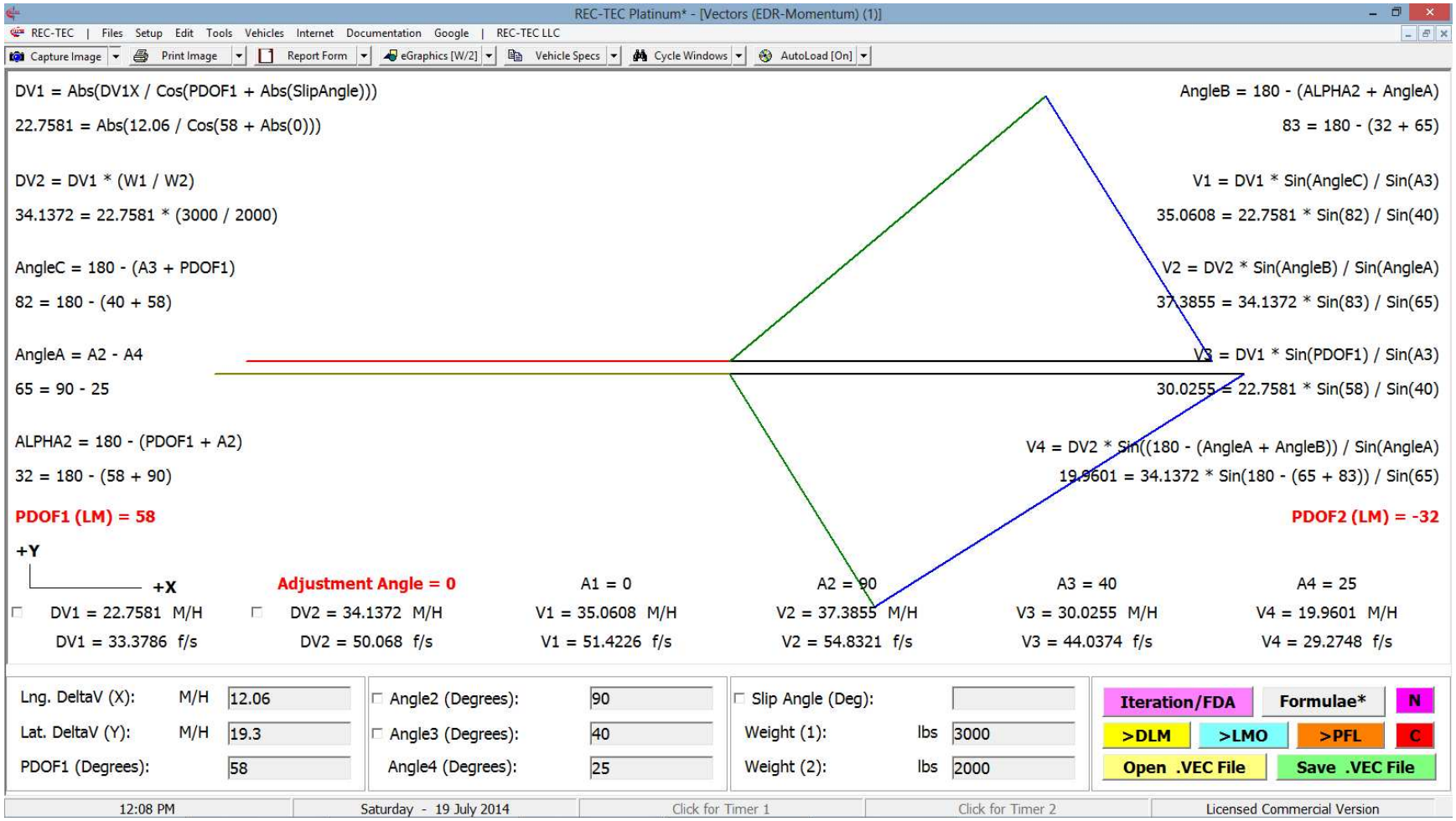
Triangle Solution for V2, V4, and AngleB
Known: AngleA = 65, PDOF2 = 32, and DV2 = 34.1372
Solution: V2 = 37.3854, V4 = 19.96, and AngleB = 83

Figure 25: Vectors (EDR-Momentum) Solution



Vectors (EDR-Momentum) Solution

Figure 26: Vectors (EDR-Momentum) Solution



Vectors (EDR-Momentum) Solution

Example 4 – Vectors (EDR-Momentum)

Note: In late 2011 this module was upgraded. The upgraded Vector module is now capable of sending its Momentum solution directly to the 360 Linear Momentum module using the **[> LMO]** button. Two Checkboxes have also been added in front of the DV1 and DV2 solutions. Placing a check in one of these boxes will bring up the Solution Diagram for the corresponding PDOF/DeltaV. Un-checking the box will return the user to the Momentum Vector diagram. Using the [Esc] key will show the Longitudinal and Lateral components used to arrive at the PDOF solution if that methodology was employed in the solution. The ToolTips (place cursor over the inputs or Checkboxes) will describe the effect these actions will have on the modules displays.

Right clicking on **Formulae*** after entering the data creates Figure 27.

Figure 27: Vectors (EDR-Momentum) Solution with Formulae*

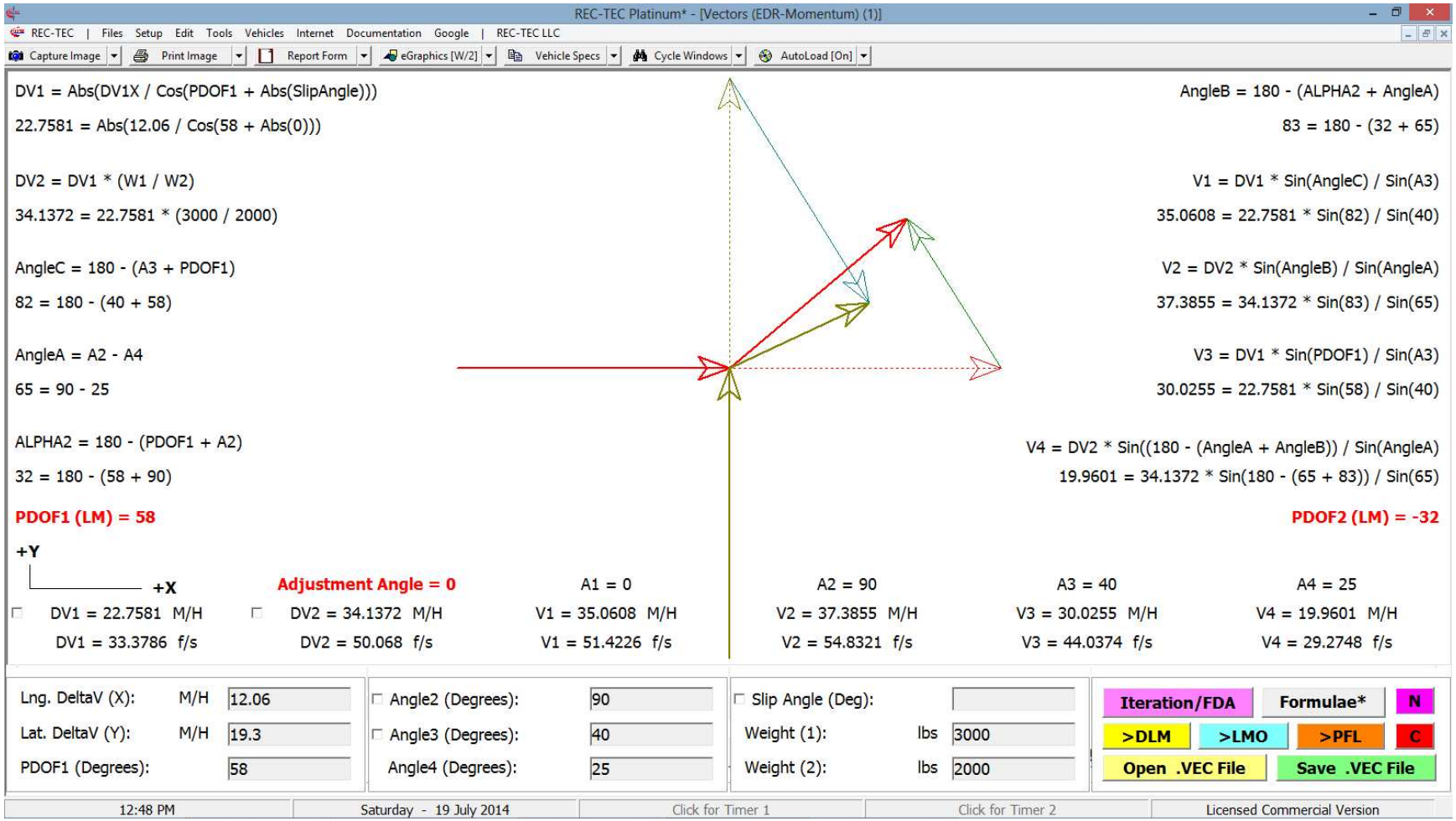


Figure 28 uses an 80-degree approach angle for Vehicle 2.

Figure 28: Vectors (EDR-Momentum) Solution - A2 = 80 degrees

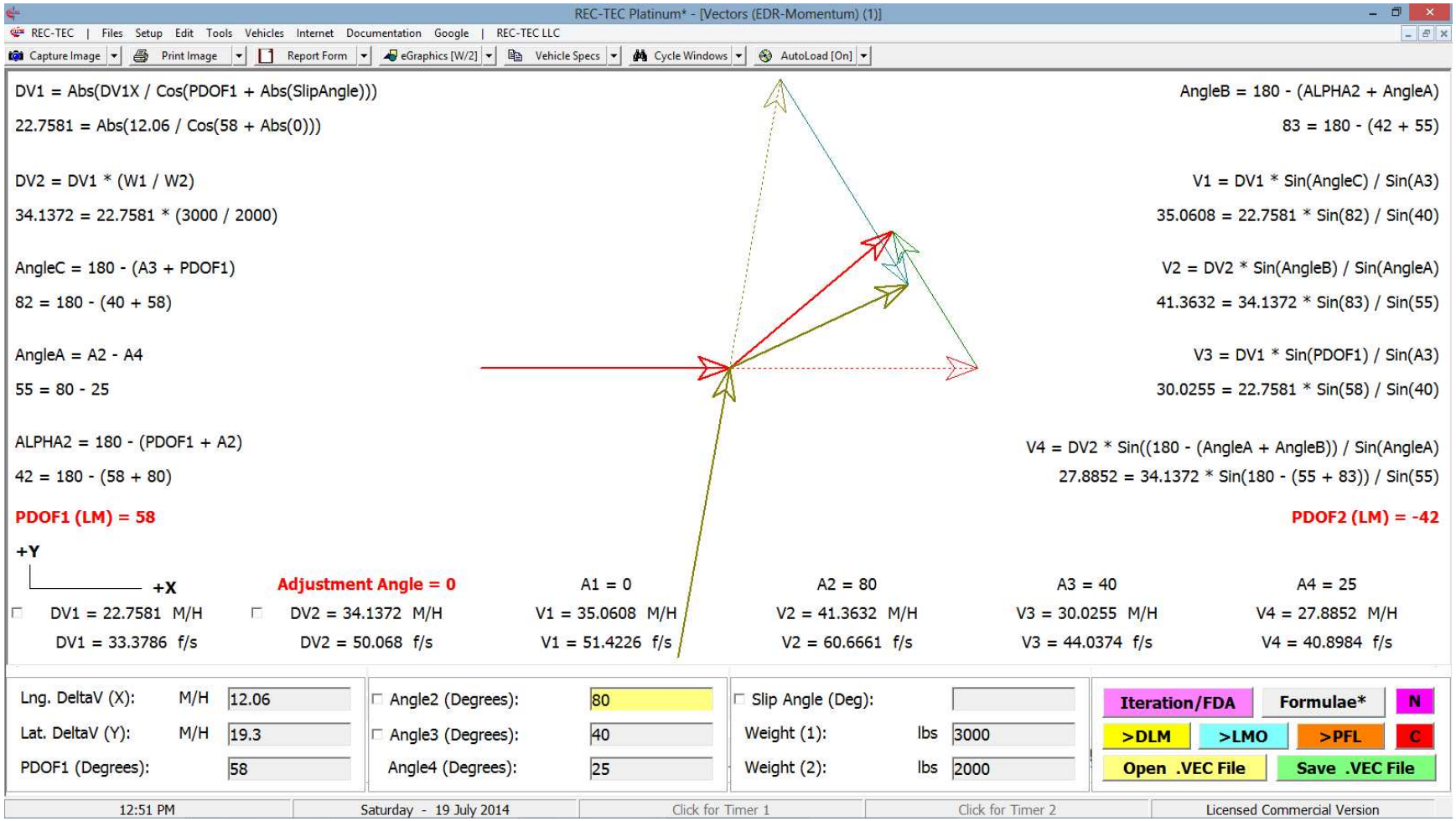


Figure 29 uses a 100-degree approach angle for Vehicle 2.

Figure 29: Vectors (EDR-Momentum) Solution - A2 = 100 degrees

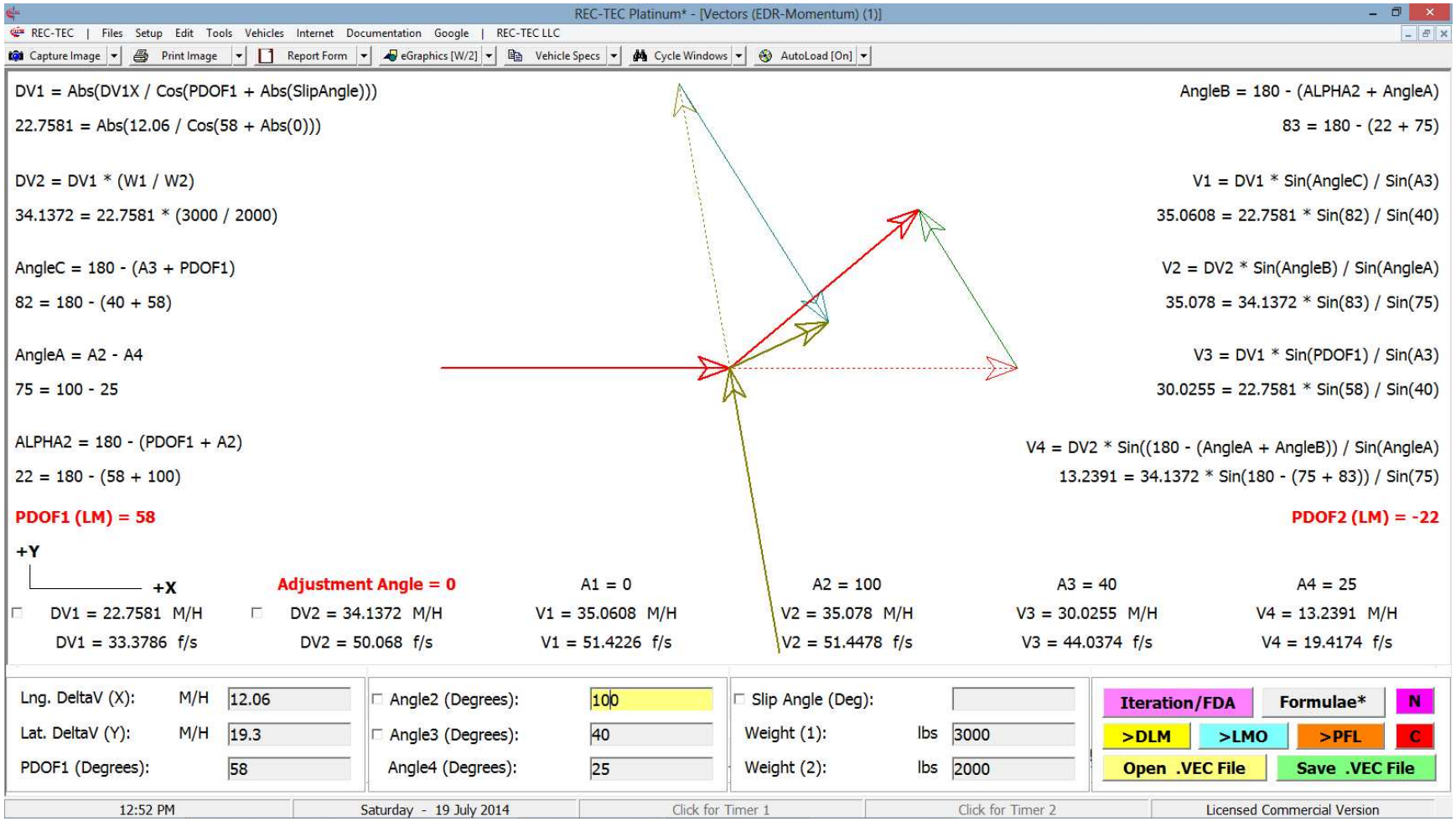


Figure 30 uses a 10-degree Slip Angle to calculate DV1 with Angle 2 = 90 degrees.

Figure 30: Vectors (EDR-Momentum) Solution - 10 degree Slip Angle with A2 = 90 degrees

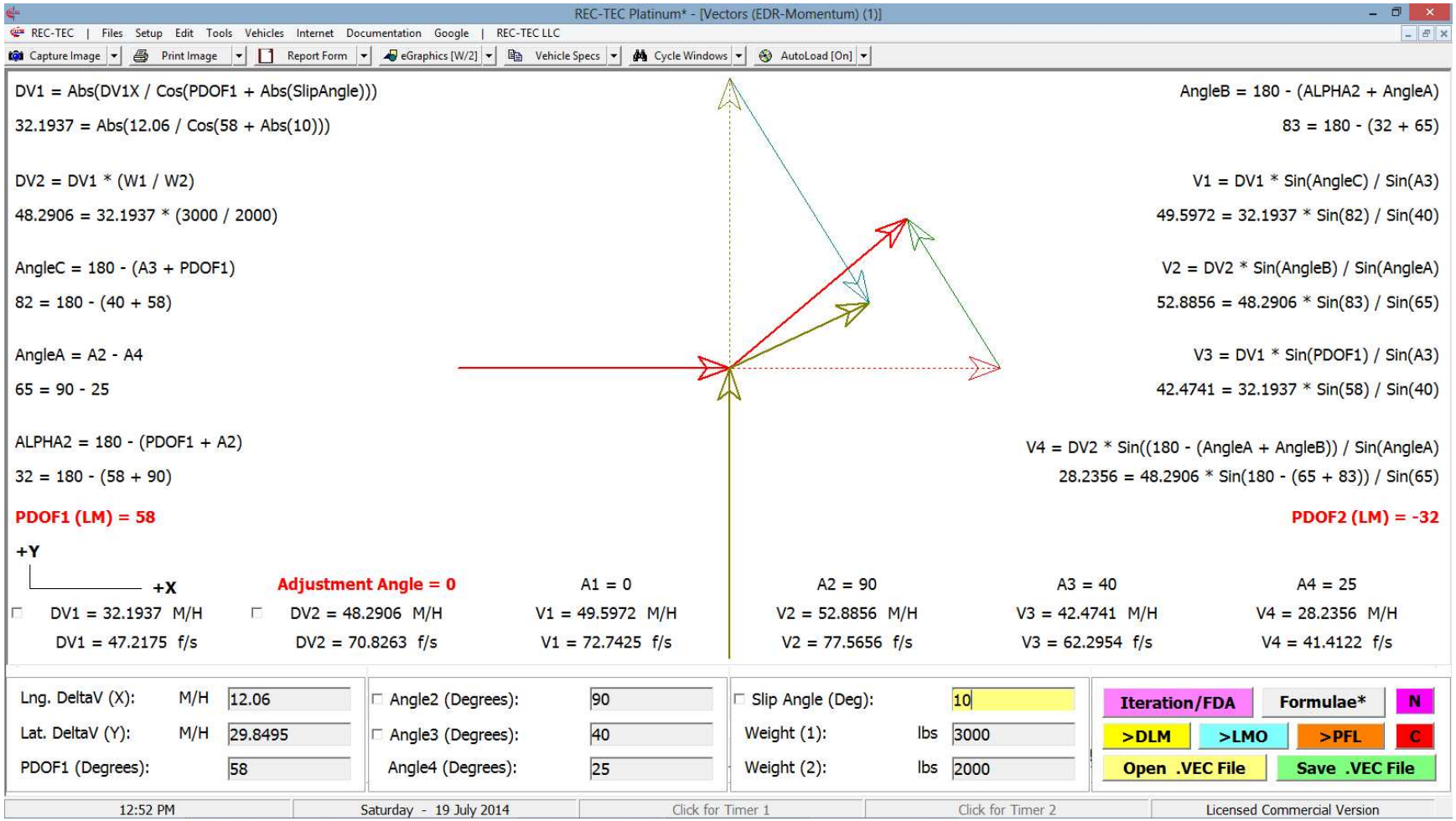


Figure 31 uses a 10-degree Slip Angle to calculate DV1 with Angle 2 at 90 degrees and with a Lateral DeltaV used to compute the PDOF angle of 58 degrees.

Figure 31: Vectors (EDR-Momentum) Delta V1 Solution - 10 degree Slip Angle with A2 = 90 degrees

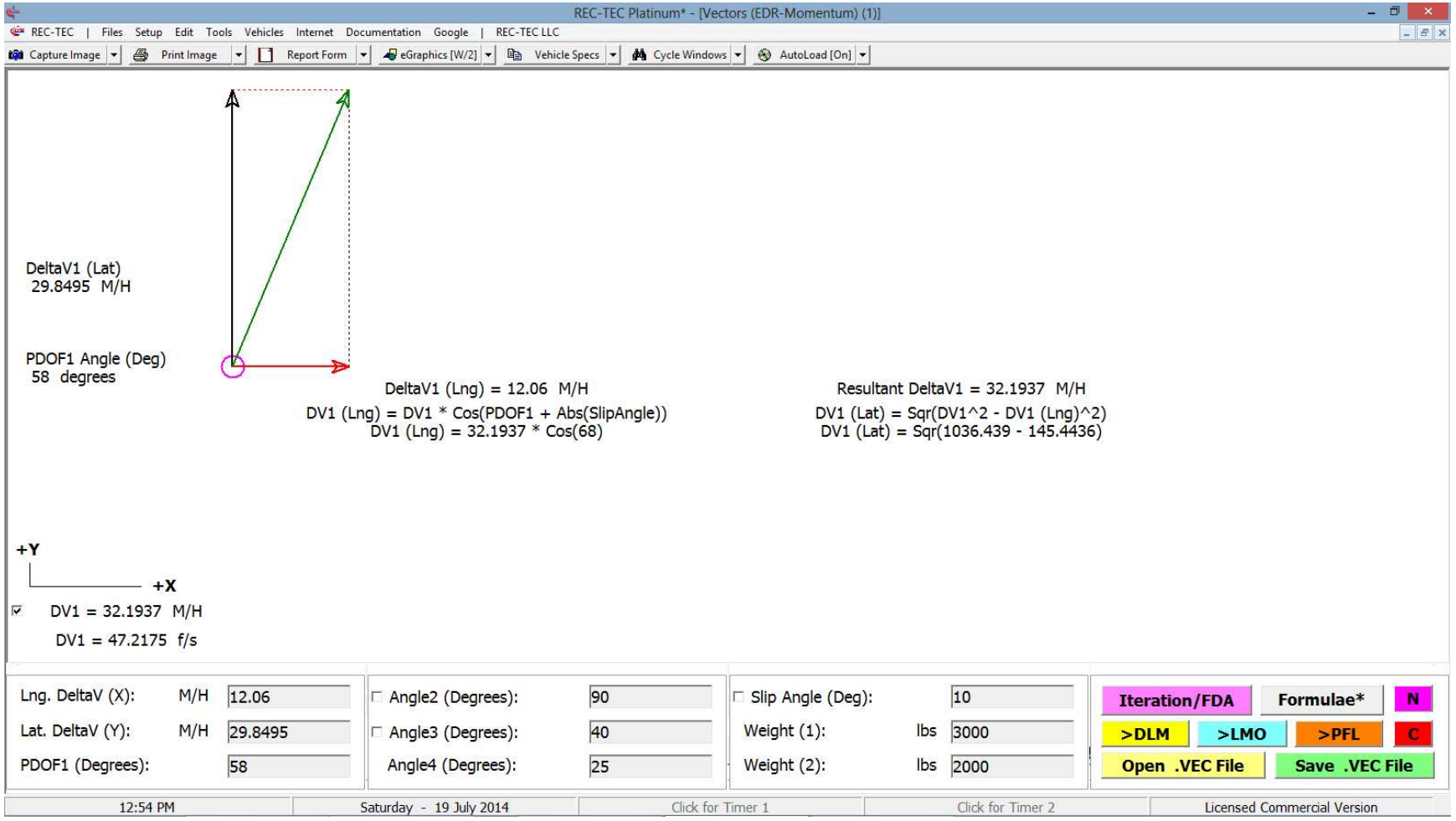
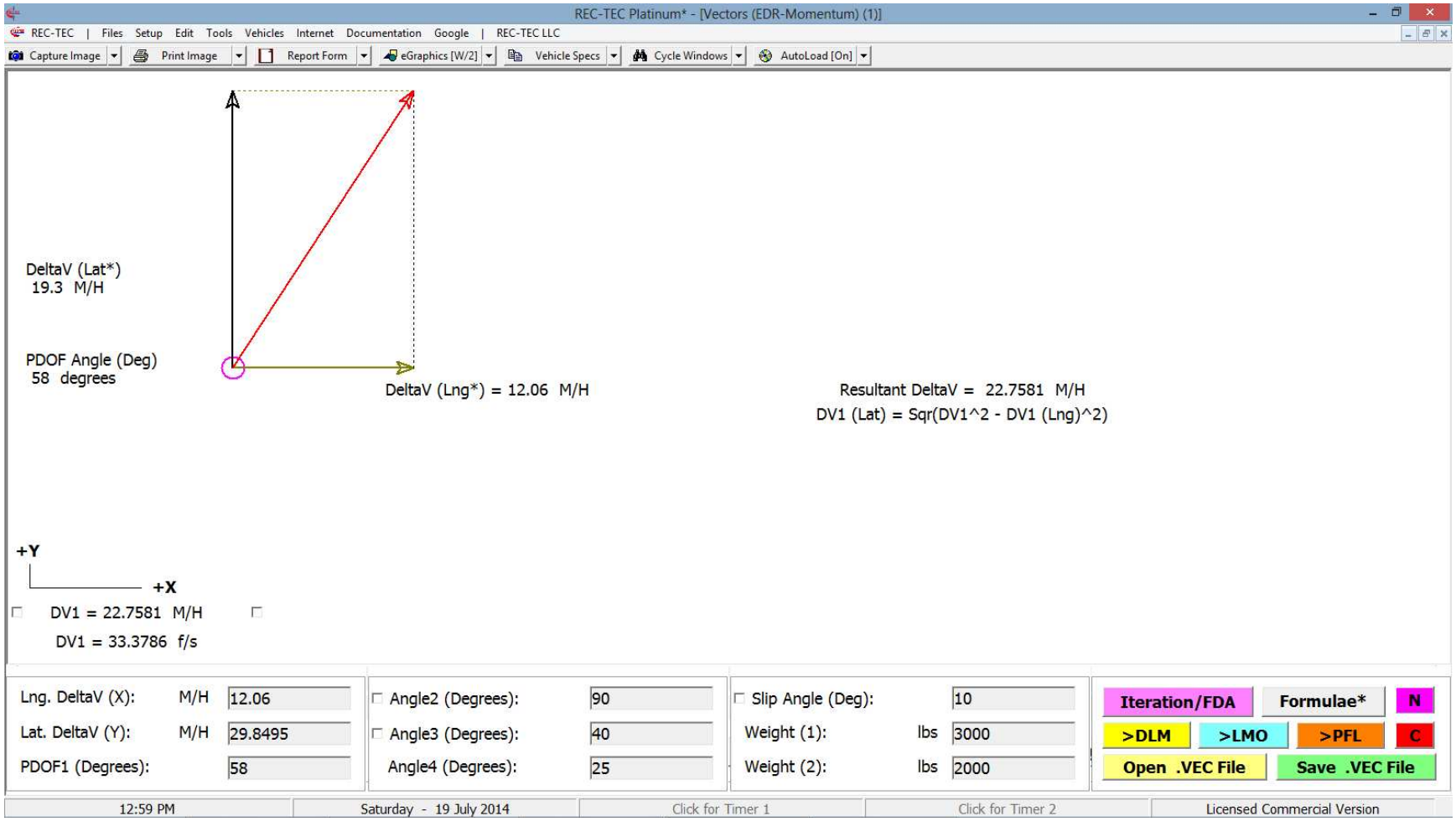


Figure 31 shows the DV1 Vector Diagram with a Check in the DV1 Checkbox. The screen shows a PDOF1 of 58 degrees along with the 10-degree slip angle used in the computations.

In order to see the information in the Vector Diagram shown in Figure 31, but without the Slip angle of 10 degrees, the 10 degrees could be removed from the Slip angle input label. This will alter the problem if the 10 degrees is not returned.

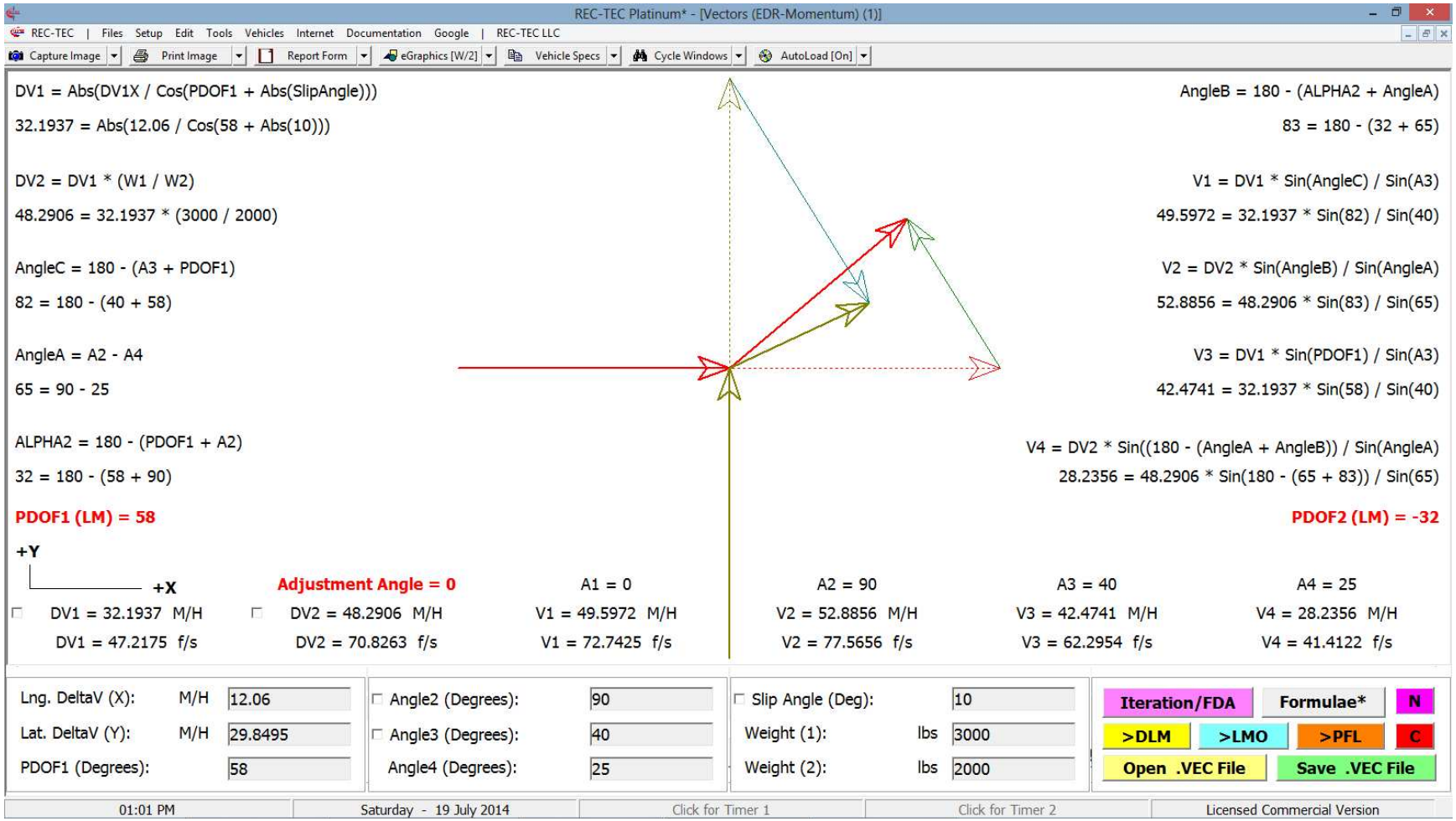
Another method is to use the [Esc] key, which brings up the data for the initial computation of 58 degrees based on the Lateral Input but does not account for the Slip angle (Figure 32). This does not put the problem at risk, as the 10 degree Slip angle has not be altered.

Figure 32: Vectors (EDR-Momentum) Delta V1 Solution - 0 degree Slip Angle with A2 = 90 degrees



A second **[Esc]** key returns the screen to the initial Momentum diagram (Figure 33).

Figure 33: Vectors (EDR-Momentum) Solution - Same as Figure 30



Advanced Solutions:

In July of 2014, this module underwent a major upgrade and a name change to **Vectors (EDR-Momentum)**, **to** more accurately reflect the capabilities of the upgraded module.

If using the AutoLoad feature when the module is first called up after the version upgrade, the only noticeable changes are:

1. The name change for the Module
2. New blocks appear over the DV2, V1, V2, V3, and V4 solution blocks
    a. Adjustment Angle
    b. A1 = 0
    c. A2 = 55
    d. A3 = 14.8641
    e. A4 = 10
3. Checkboxes are now in front of Angle2, Angle3 and Slip Angle
4. The 1<>2 button is no more.

All remaining Examples will use the following data except as noted:

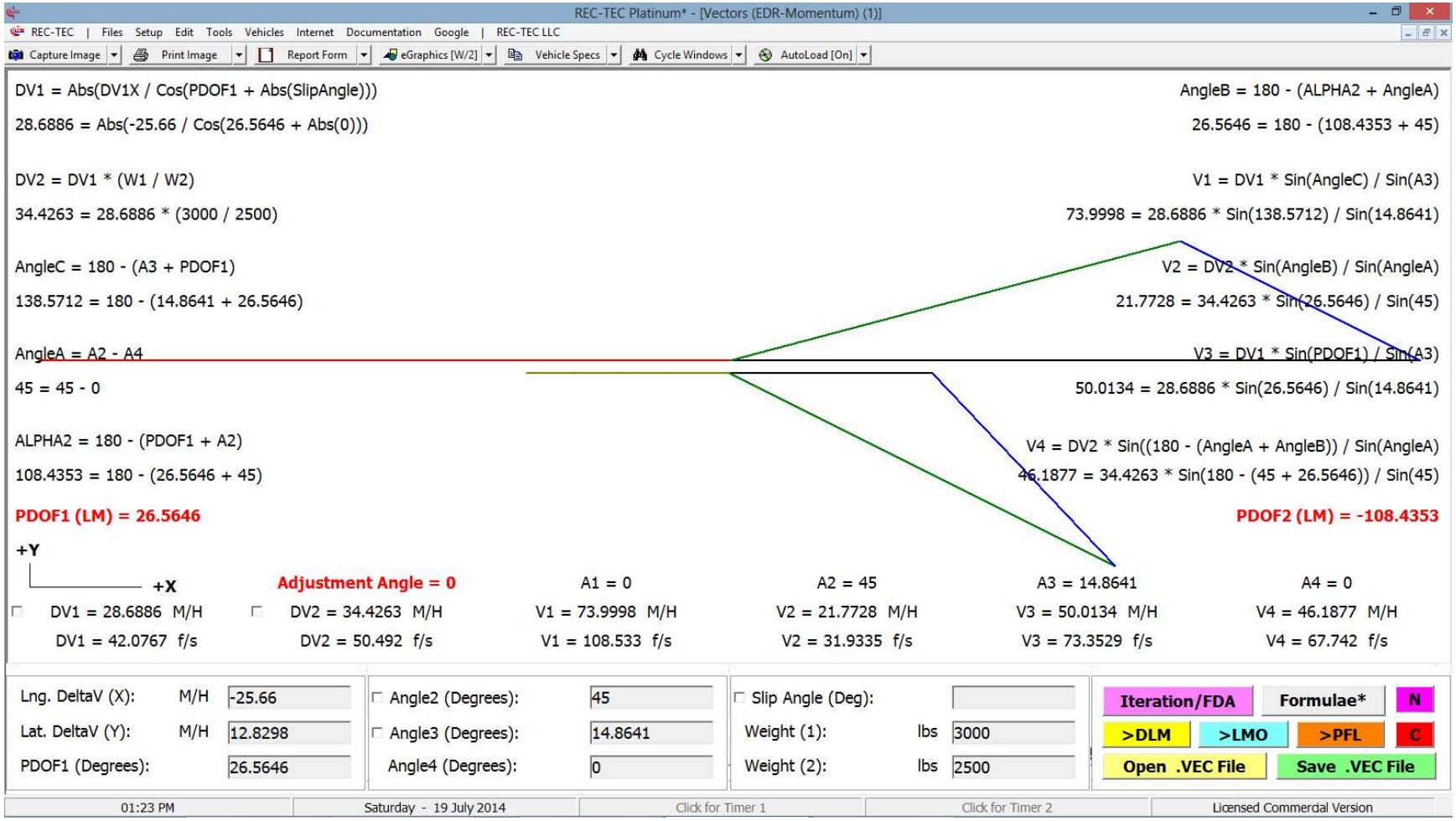
Longitudinal DeltaV = -25.66 M/H
Lateral Delta V = 12.8298 M/H
Angle 1 = 0 degrees
Slip Angle = 0 degrees
Adjustment Angle = 0 Degrees or as Shown
Weight 1 = 3000
Weight 2 = 2500

Angle 2 = As Shown
Angle 3 = As Shown
Angle 4 = As Shown

A tour of the new features in the upgrade will start with Clicking on the Formulae* button.

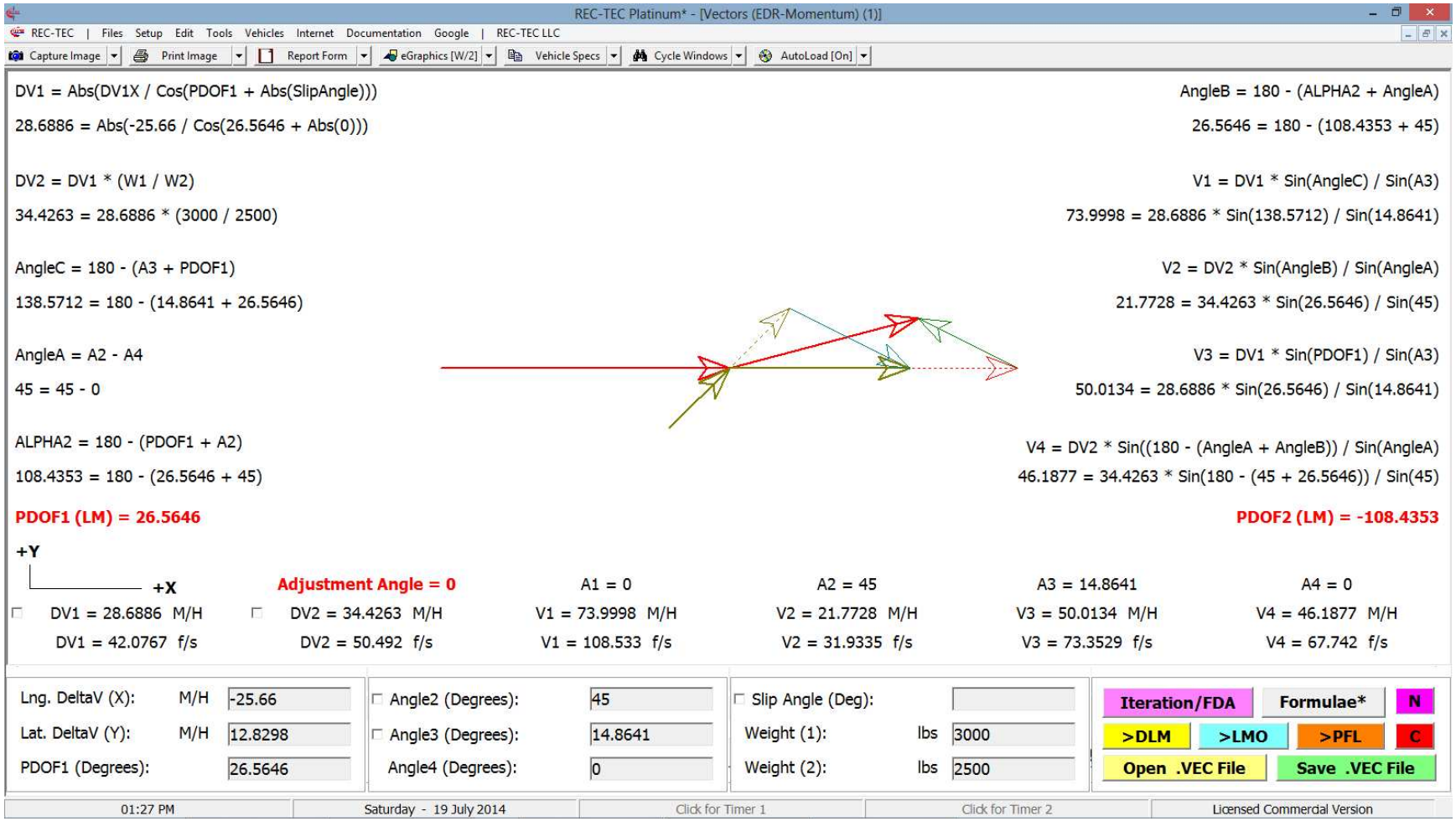
At first it may look like the Graphics are incorrect. This is the Graphics configuration for the LM-PDOF Lite module shown on the Vector screen.

Figure 34: Vectors (EDR-Momentum) Solution - Alternate Graphics



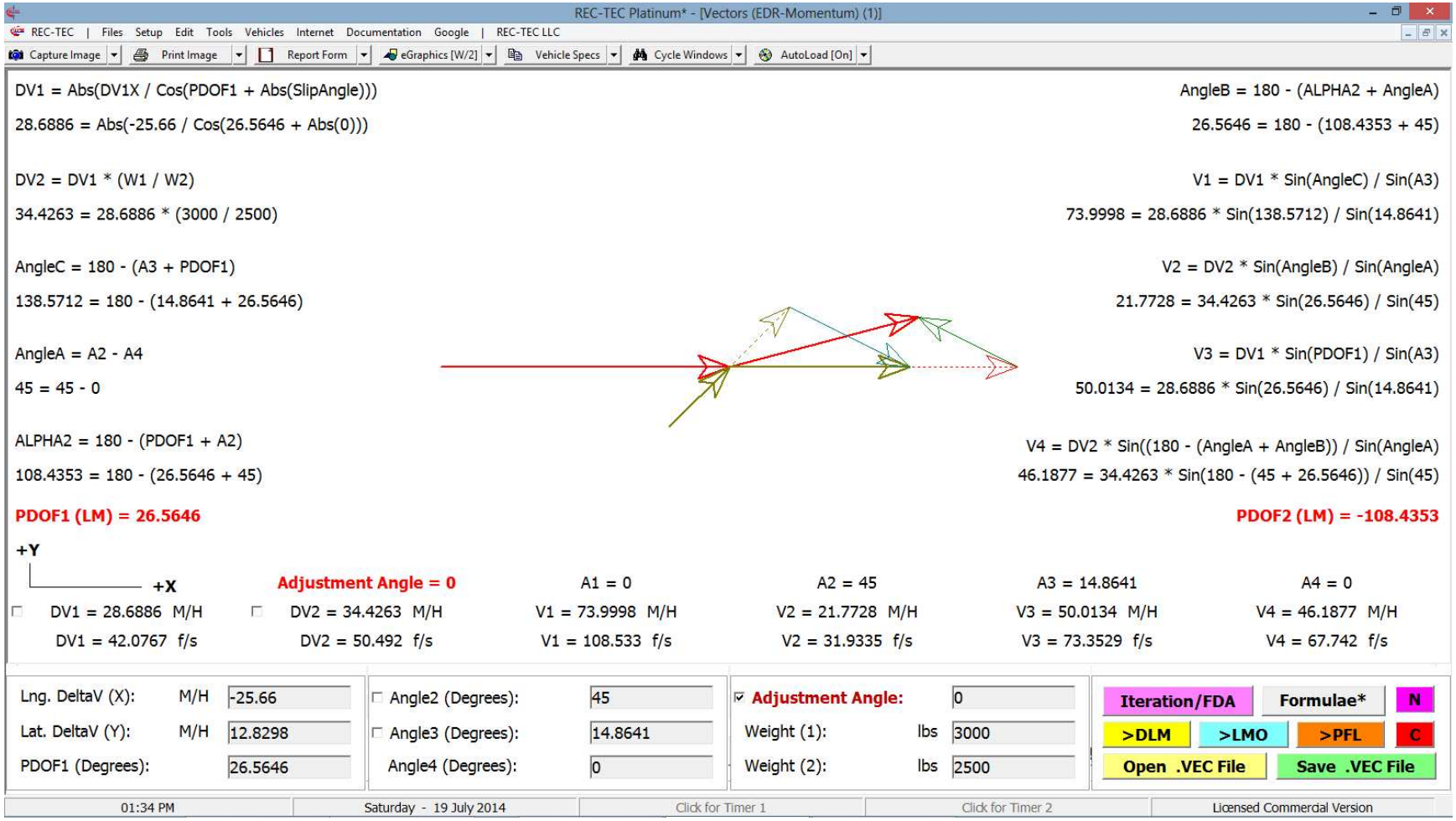
Right clicking the mouse will restore the more familiar 360 LM configuration.

Figure 35: Vectors (EDR-Momentum) Solution - 360 LM Graphics



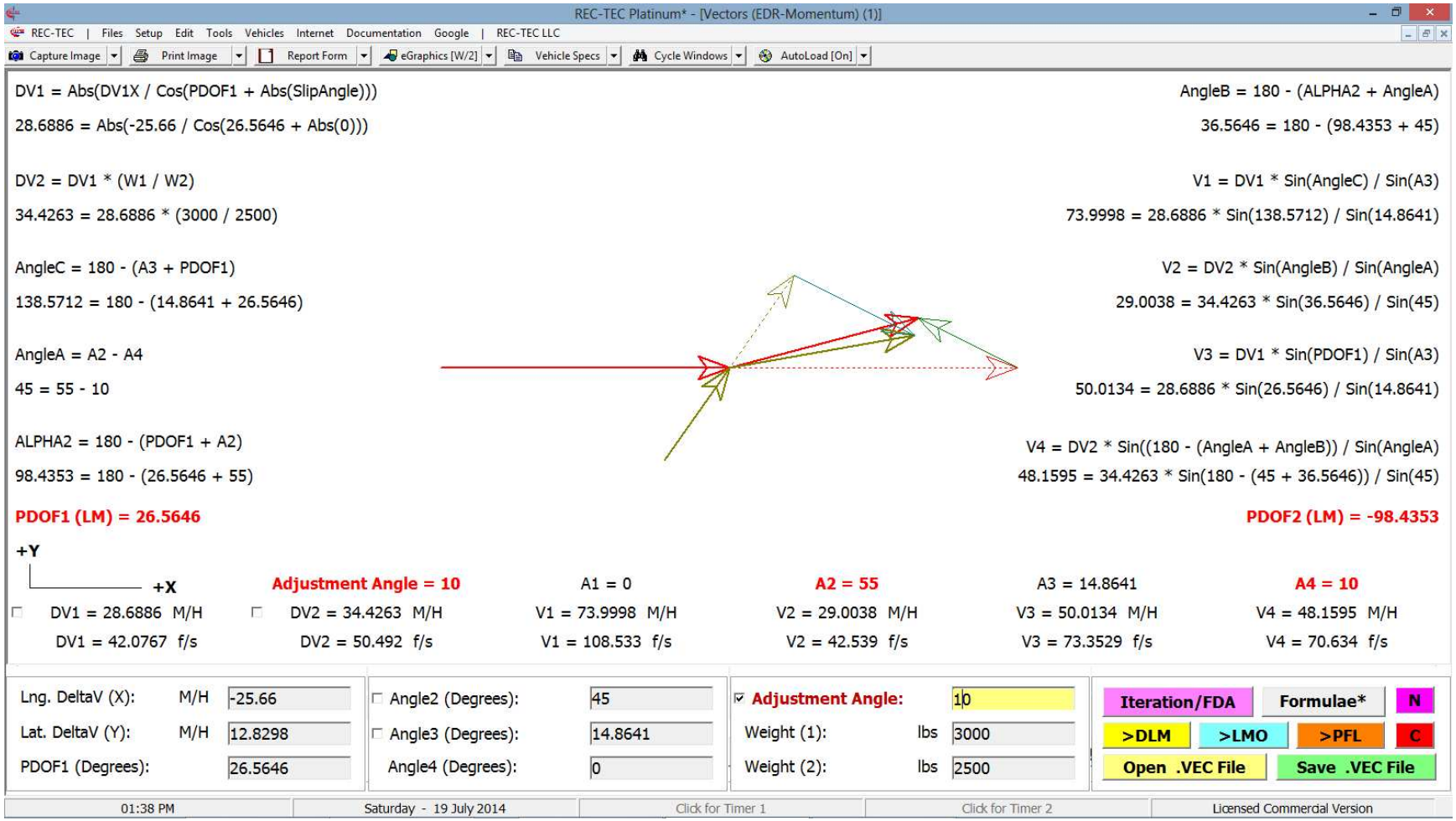
Time to show where the Adjustment Angle comes from. Notice the Checkbox in front of Slip Angle? Click on it and Slip Angle changes to Adjustment Angle and you will notice it is set to 0.

Figure 36: Vectors (EDR-Momentum) Solution - Adjustment Angle



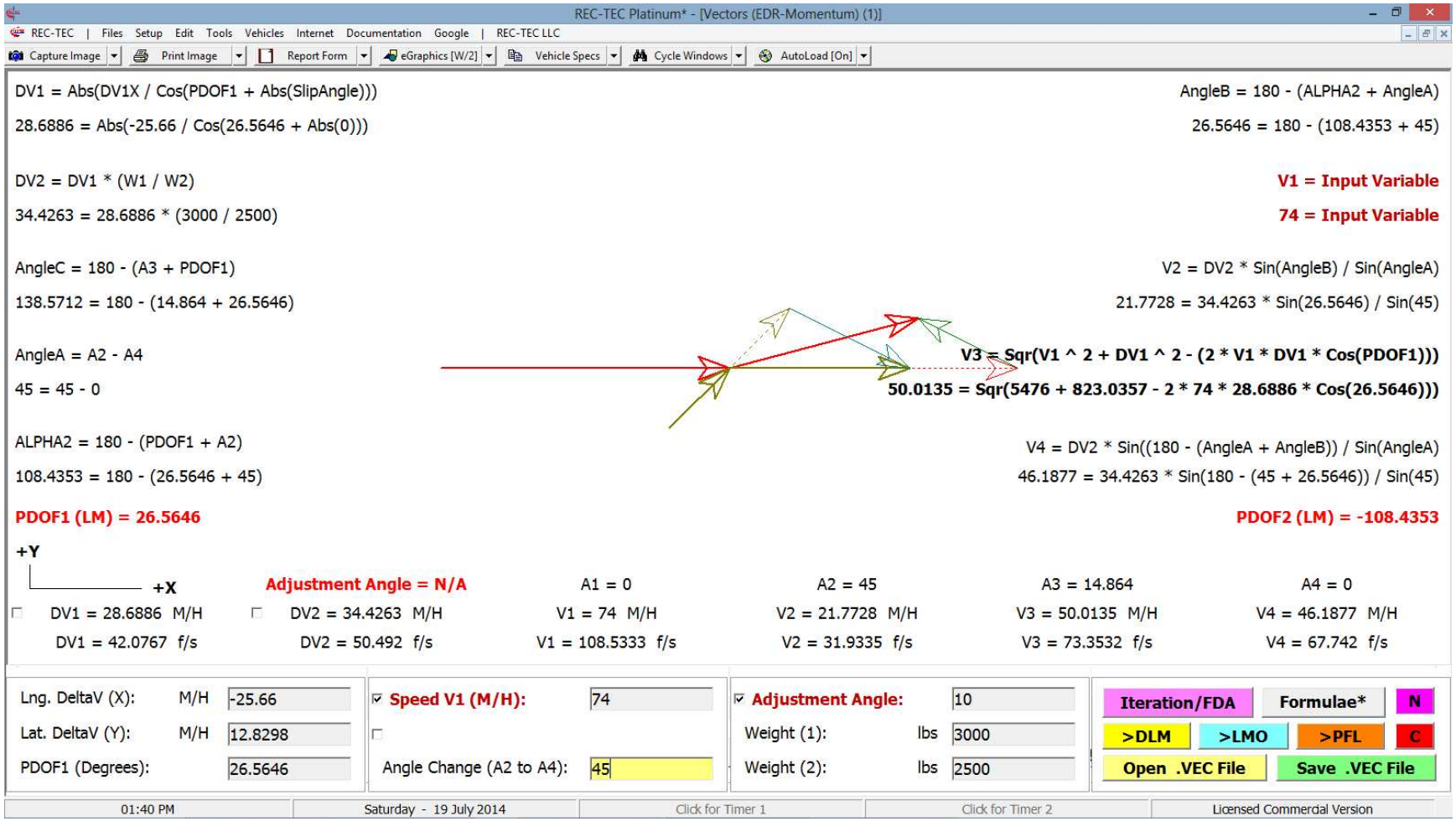
Change the Adjustment Angle to 10 degrees. Notice that it has not changed the base angles (below the Graphics Screen) to the Angles shown in red on the Graphics screen. We will come back to this later as it is a Major feature critical to the accuracy of the rest of the upgrades.

Figure 37: Vectors (EDR-Momentum) Solution - Adjustment Angle = 10 degrees



Notice that there is a Checkbox in front of the Angle2 input line below the Graphics.

Figure 38: Vectors (EDR-Momentum) Solution - V1 Speed Input

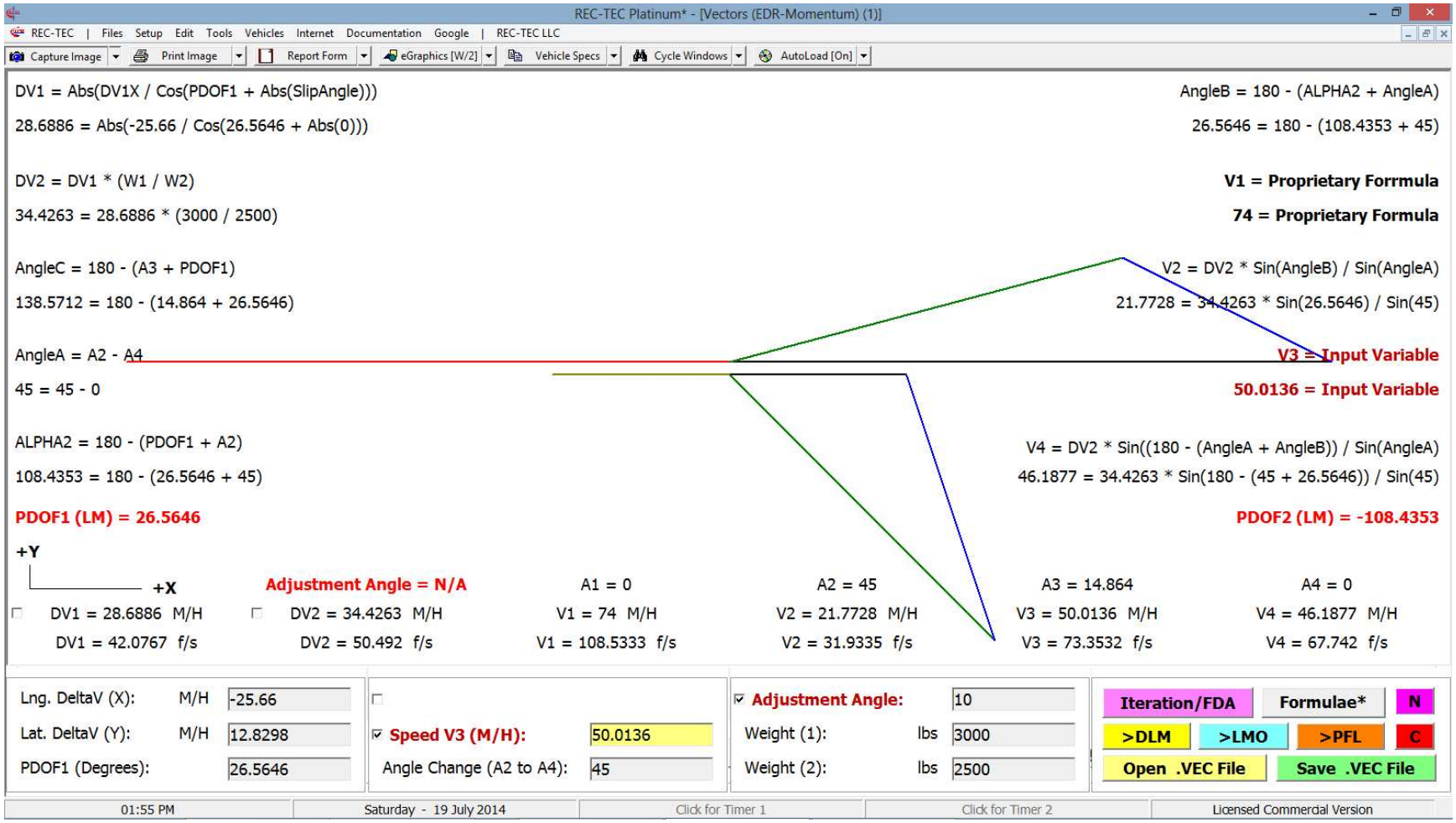


Clicking on that Checkbox brings up an Input for the V1 speed which should show 74 M/H if that is the Primary input. A keen eye will notice that some of the Formulae have changed along with the data. The Input block asking for Angle3 has disappeared but the Checkbox remains. The Angle4 input has changed to "Angle Change (A2 to A4)" and now shows 45 instead of 0. Note also that in the Formulae on the Graphics screen, V1 is described as an Input of 74.

The Adjustment Angle now shows "N/A" as angles are not required to perform these computations so there are no angles to adjust. Angles A2, A3, and A4 are computed by the program and inserted into the respective Angle Inputs for future analysis. The Angles shown on the Graphics screen do not include the Angle Adjustment.

Notice the Checkbox in front of where the Angle2 input line was below the Graphics. Left click on it.

Figure 39: Vectors (EDR-Momentum) Solution - V3 Speed Input



Clicking on that Checkbox brings up an Input for the V3 speed which should show 50.0136 M/H if that is the Primary input. A keen eye will notice that some of the Formulae have again changed along with the data. The Input block asking for Angle2 has disappeared but the Checkbox remains. The Angle4 input still shows the change to "Angle Change (A2 to A4)" and still shows 45 instead of 0. Note also that in the Formulae on the Graphics screen, V3 is described as an Input of 50.0136.

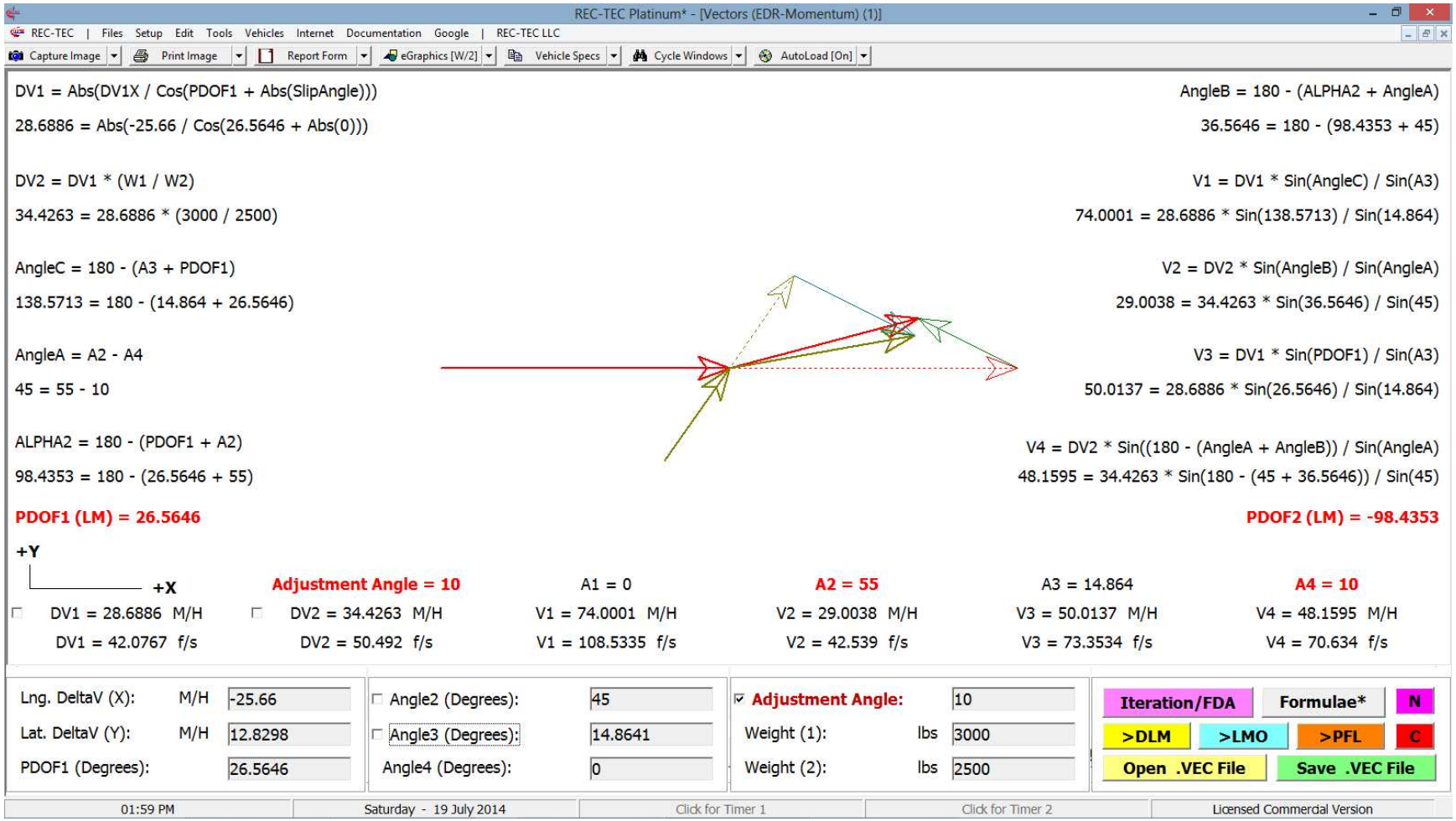
V1 may show 73.9999 (truncation) and Proprietary Formula. The specific formula is not needed as it can be proved correct using a variety of methods.

So how did we get V3 and Why? Tackling the "Why" question first, because maybe the EDR did not collect the V1 information and we were unsure of other methods of obtaining V1. If we are able to get a good post-impact speed at separation for V3, it is always a good idea to run it as a cross check. Why not bolster your own case whenever possible? As for the "How" aspect, one good way is to do a skid to stop computation if possible, but this is just one way of many that may present themselves.

The Adjustment Angle now shows "N/A" as angles are not required to perform these computations therefore there are no angles to adjust. Angles A2, A3, and A4 are computed by the program and inserted into the respective Angle Inputs for future analysis. The Angles shown on the Graphics screen do not include the Angle Adjustment.

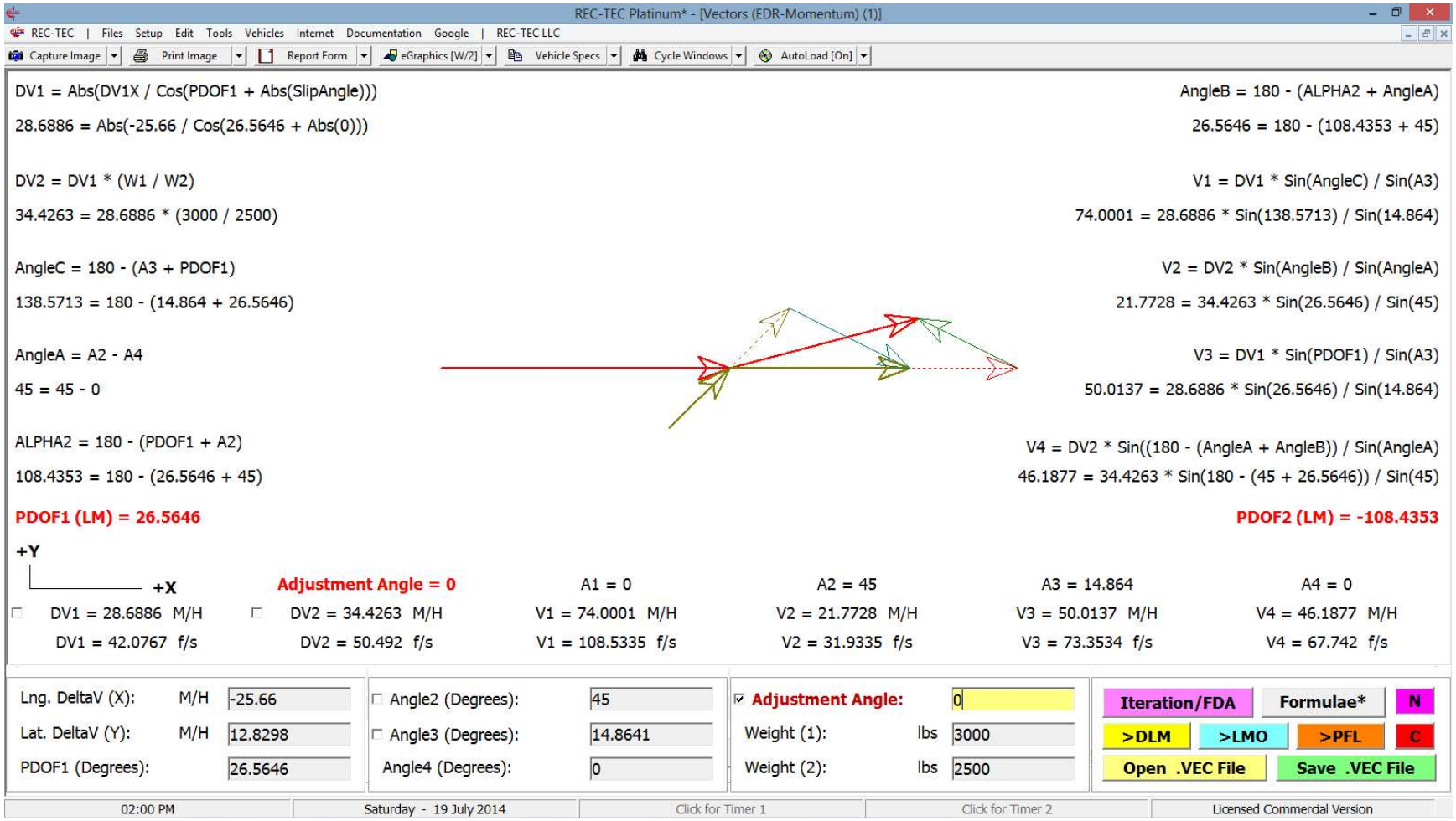
Uncheck the V3 Checkbox.

Figure 40: Vectors (EDR-Momentum) Solution - V3 Speed Input with Adjustments



It is time to examine Adjustment Angle. Change it to Zero.

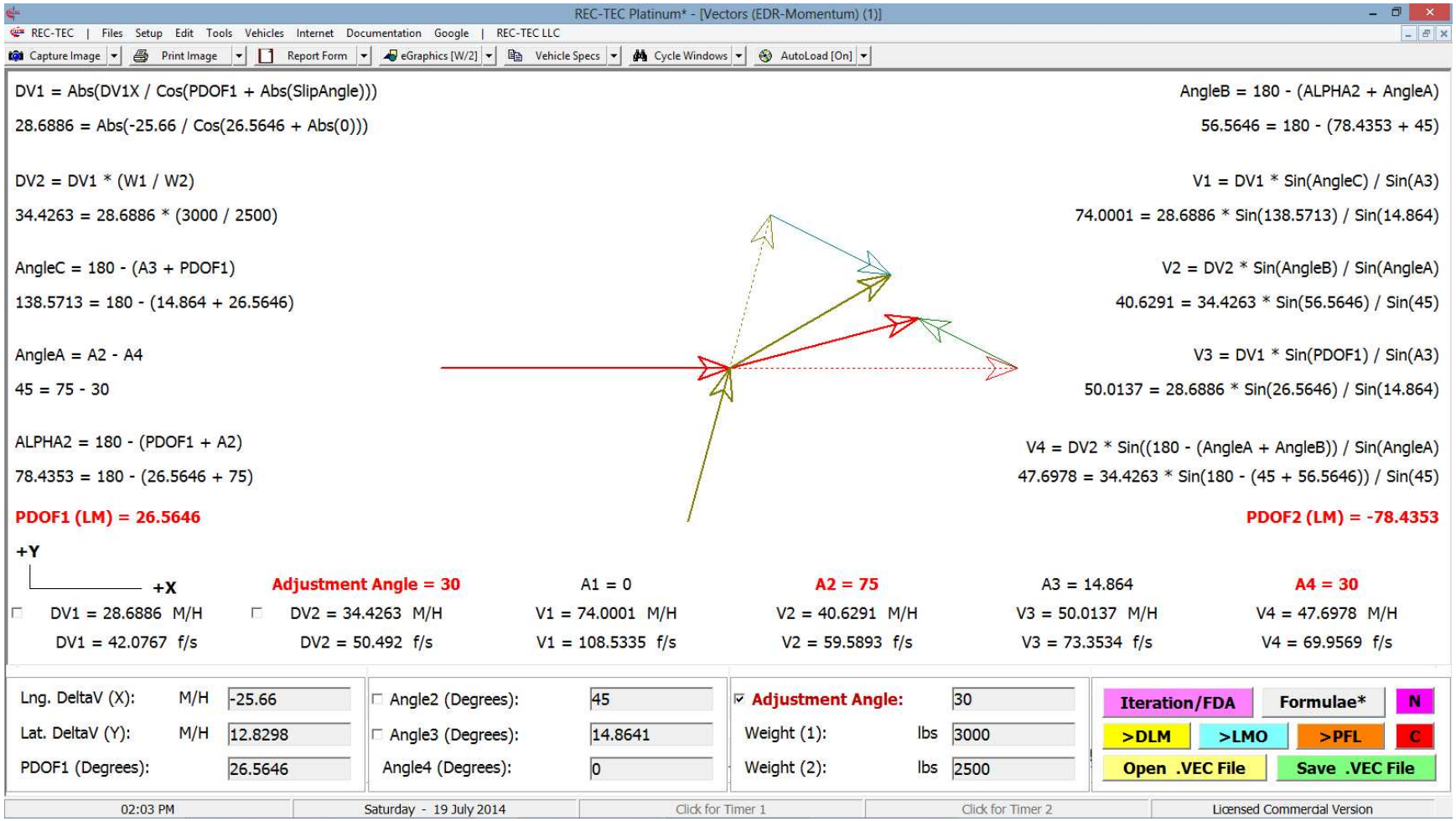
Figure 41: Vectors (EDR-Momentum) Solution - V3 Speed Input with Adjustment Angle = 0 degrees



Notice that the Angles on the Graphics section have changed. Look what also changed; V2 and V4. V1 and V3 have not changed.

As long as we are examining the Adjustment Angle, make it 30.

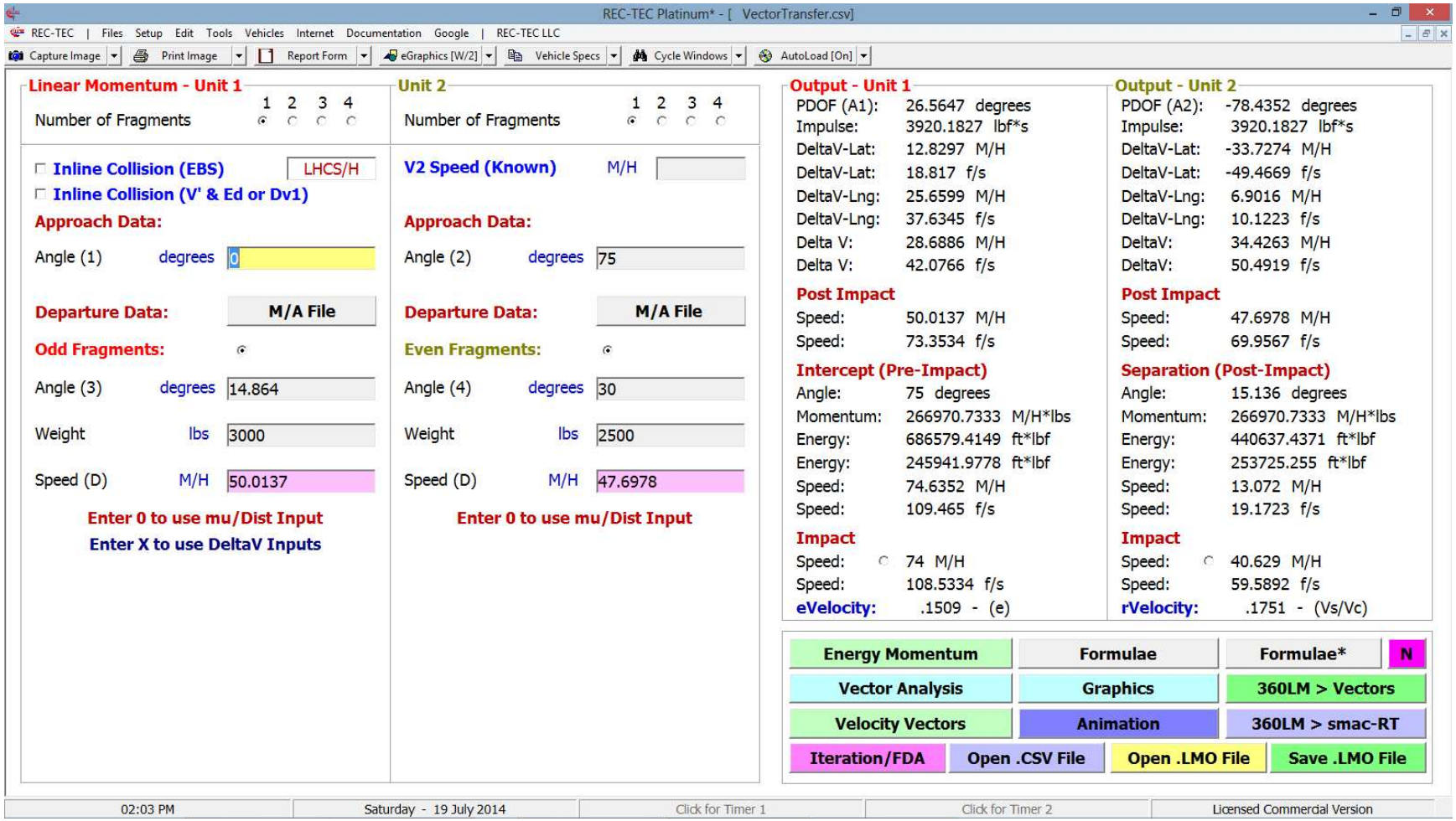
Figure 42: Vectors (EDR-Momentum) Solution - V3 Speed Input with Adjustment Angle = 30 degrees



It looks correct. Both vehicles depart of the Cone of Departure. It looks like the centers of mass have crossed.

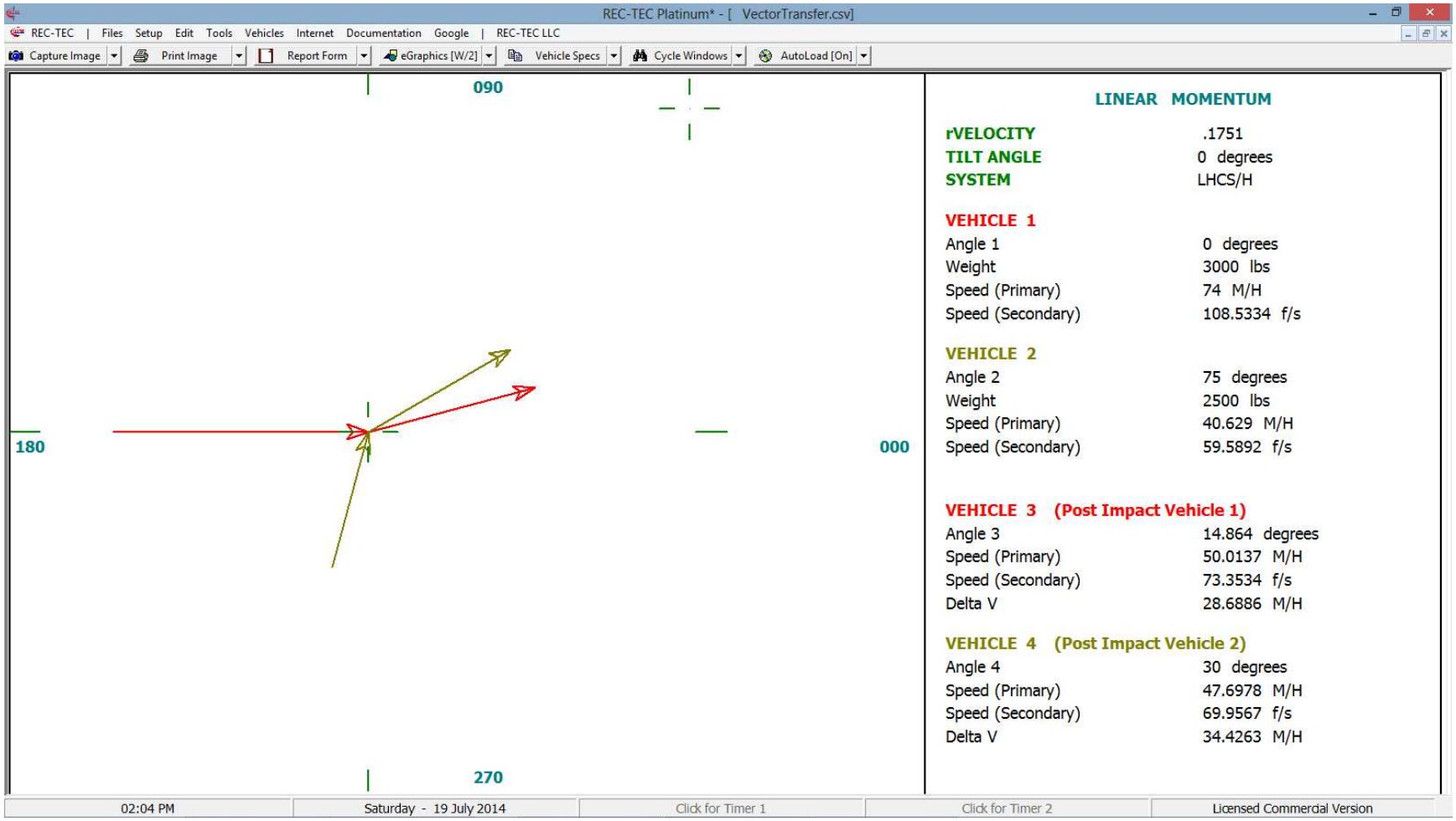
Left click on the light blue >LMO button to send this to 360 Linear Momentum for further analysis.

Figure 43: Vector Solution transferred to 360 Linear Momentum



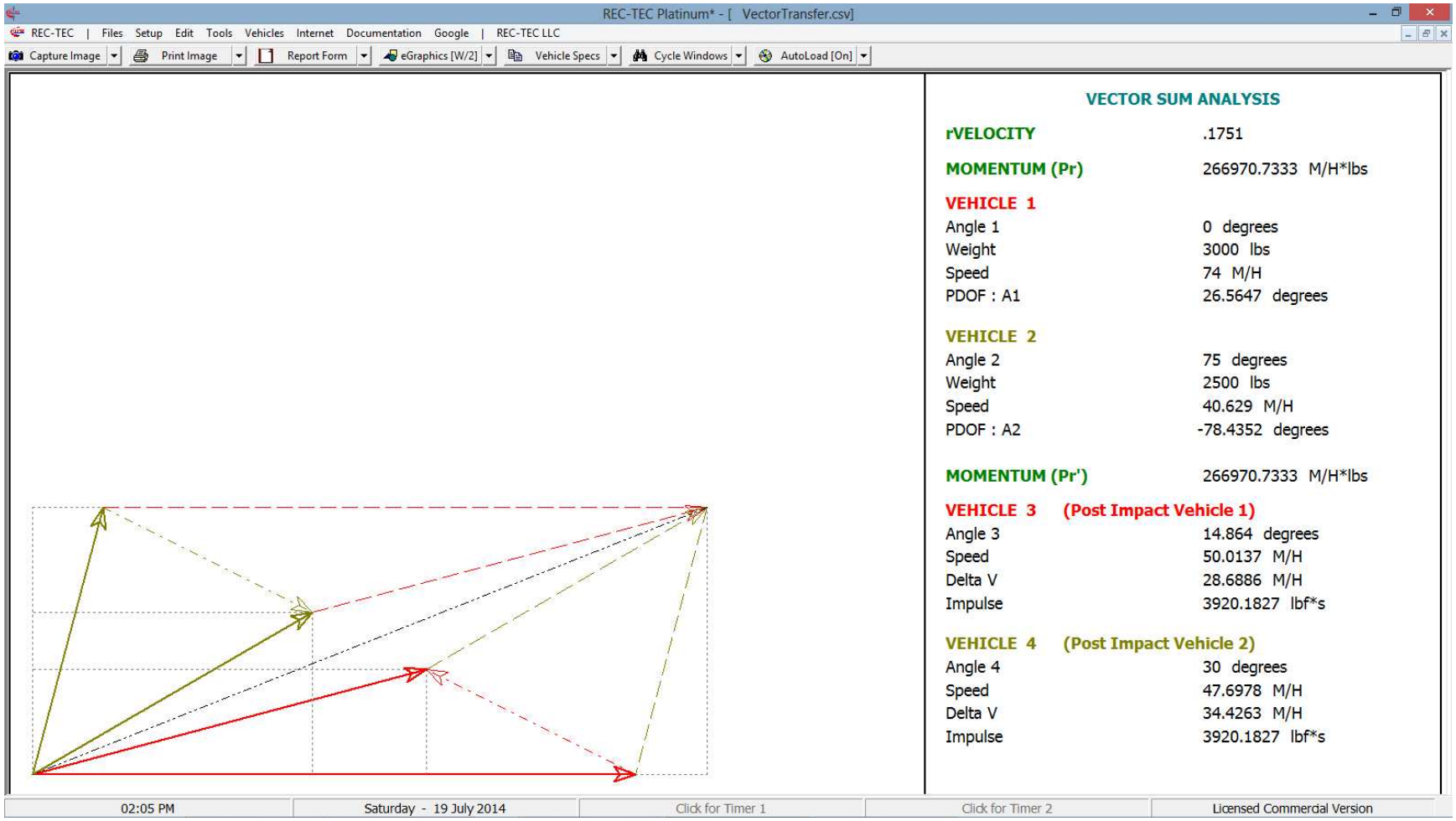
It looks like the same problem and we are getting the same answers.

Figure 44: Vector Solution transferred to 360 Linear Momentum Graphics



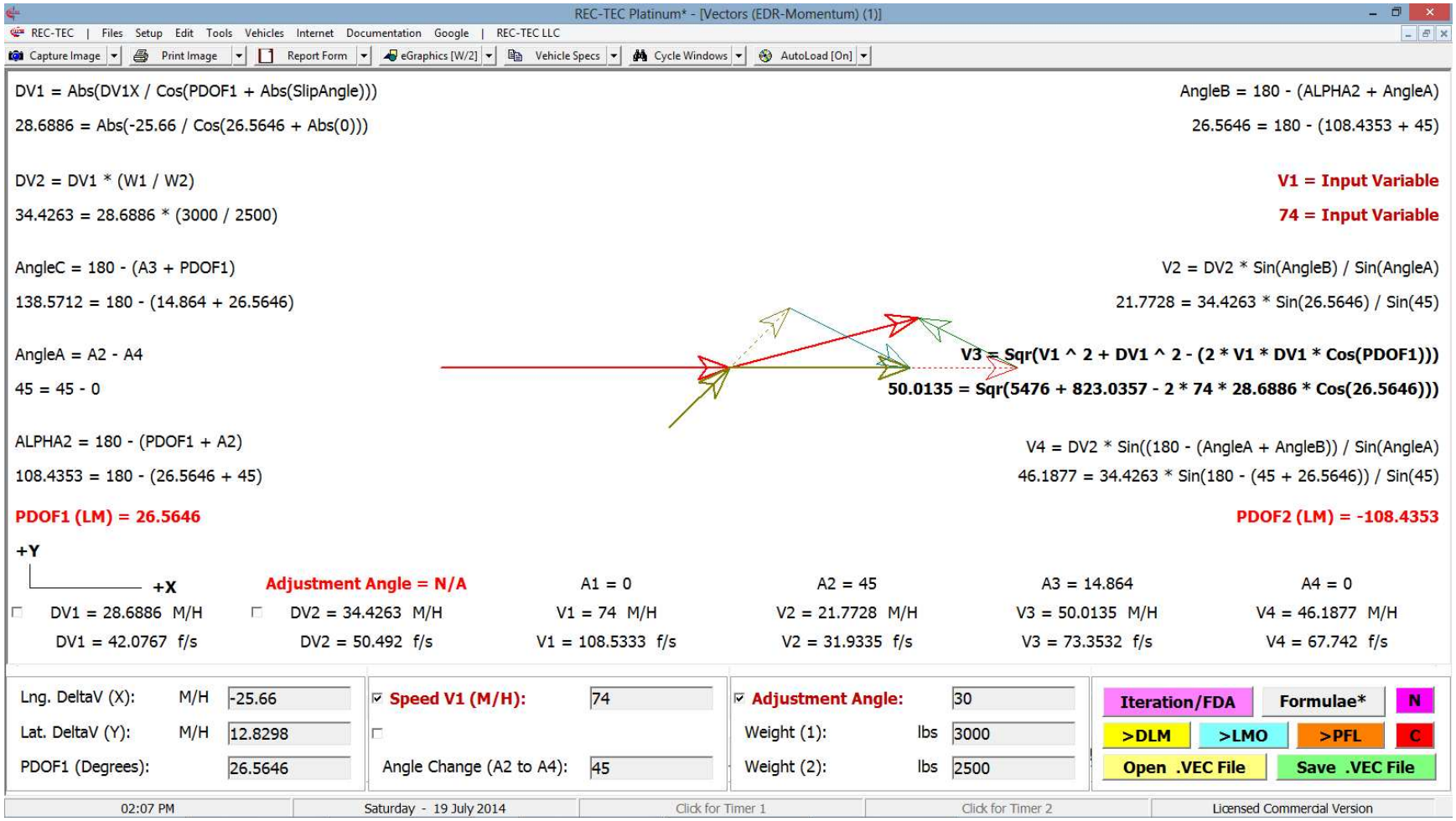
Graphics look OK. Let's examine it using Vector Sum Analysis.

Figure 45: Vector Solution transferred to 360 Linear Momentum Vector Analysis



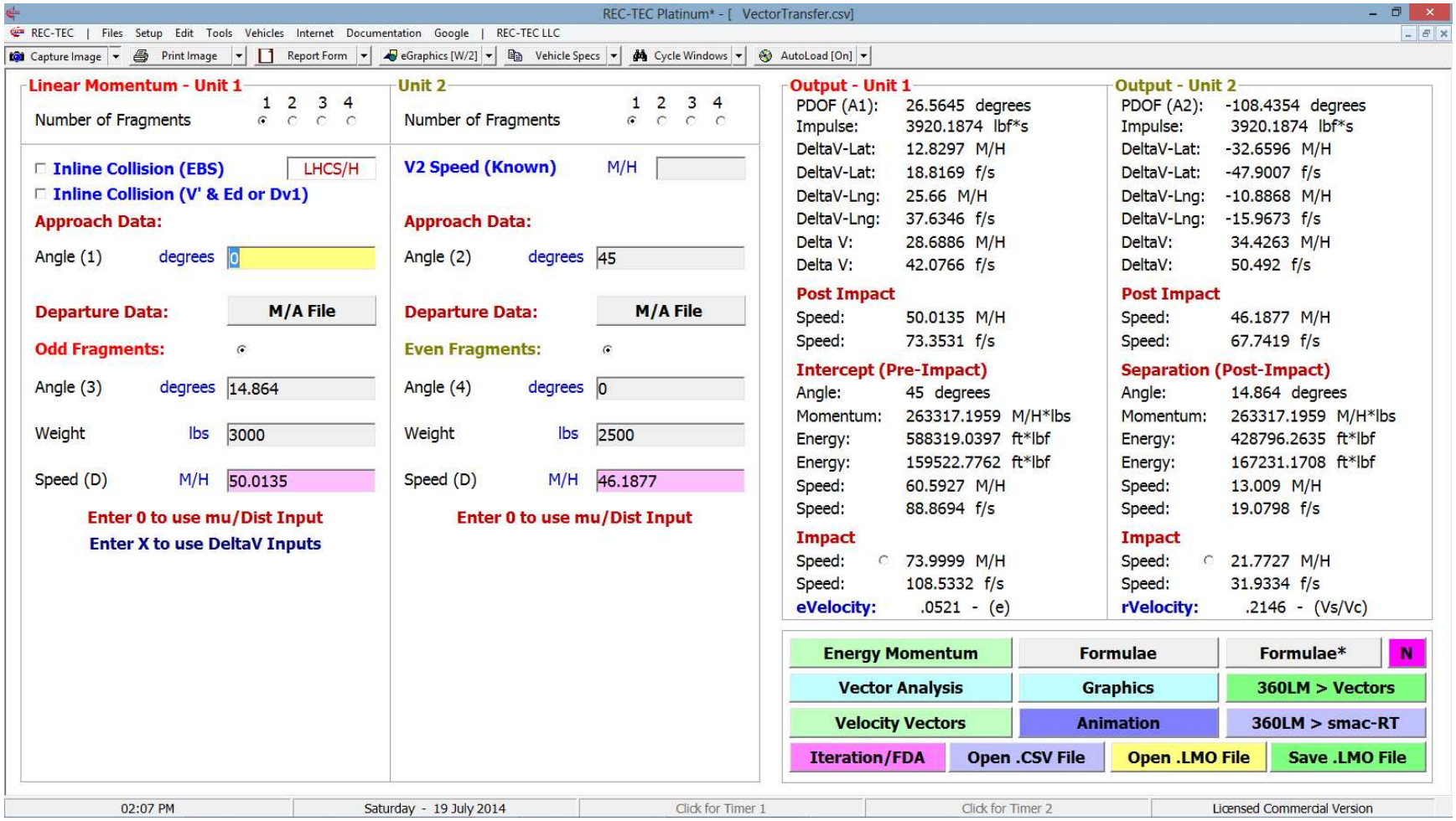
Everything seems to be just fine here too. Looks like we have a winner. Before getting overconfident, go back to the Vector Screen, and select the V1 Speed Checkbox.

Figure 46: Vectors (EDR-Momentum) Solution - V1 Speed Input with Adjustment Angle = 0 degrees



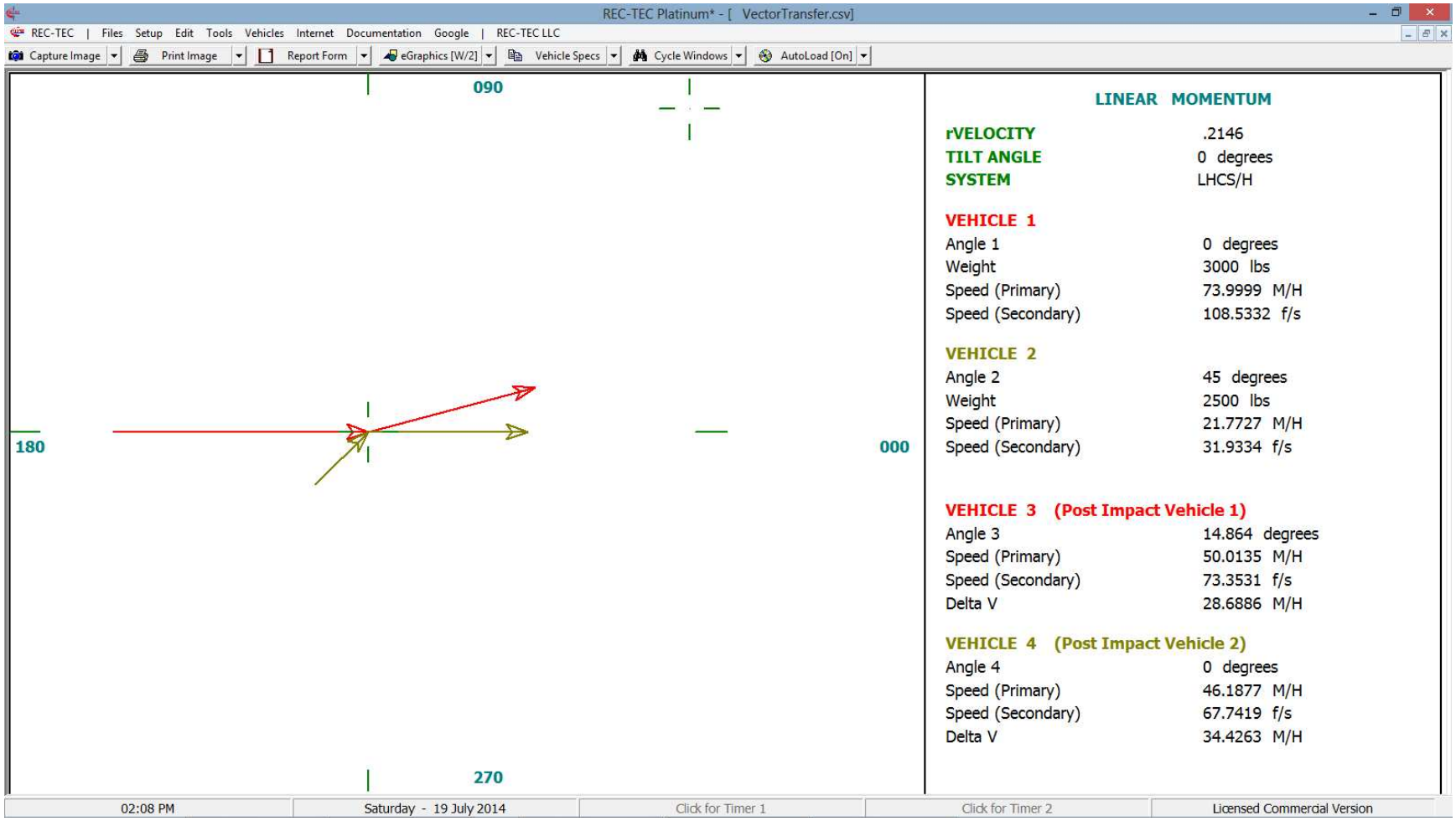
The numbers are back to the original numbers for the V1 Speed Input. Transfer this to 360 Linear Momentum.

Figure 47: Vector Solution transferred to 360 Linear Momentum



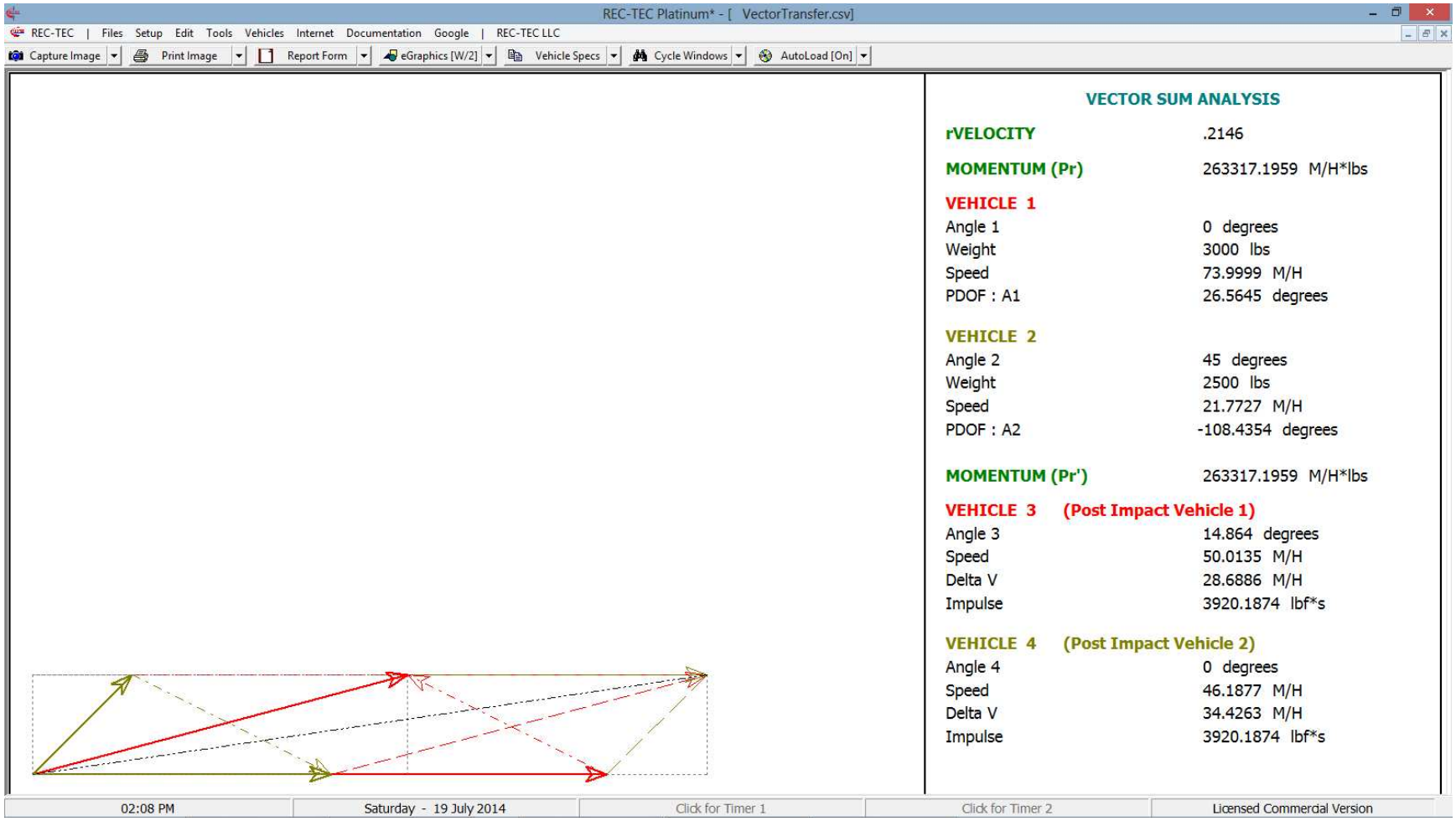
The problem and answers appear to be the same as on the Vector screen. Go to Graphics.

Figure 48: Vector Solution transferred to 360 Linear Momentum Graphics



That looks OK. Now go to Vector Sum Analysis.

Figure 49: Vector Solution transferred to 360 Linear Momentum Vector Analysis



Looks different than Figure 45, but both resultant vectors overlap each other and they meet at the same point. It must be OK.

How can they both be OK and be so different? It is because the answers are different. Look at the V2 and V4 speeds in both problems. They are very different.

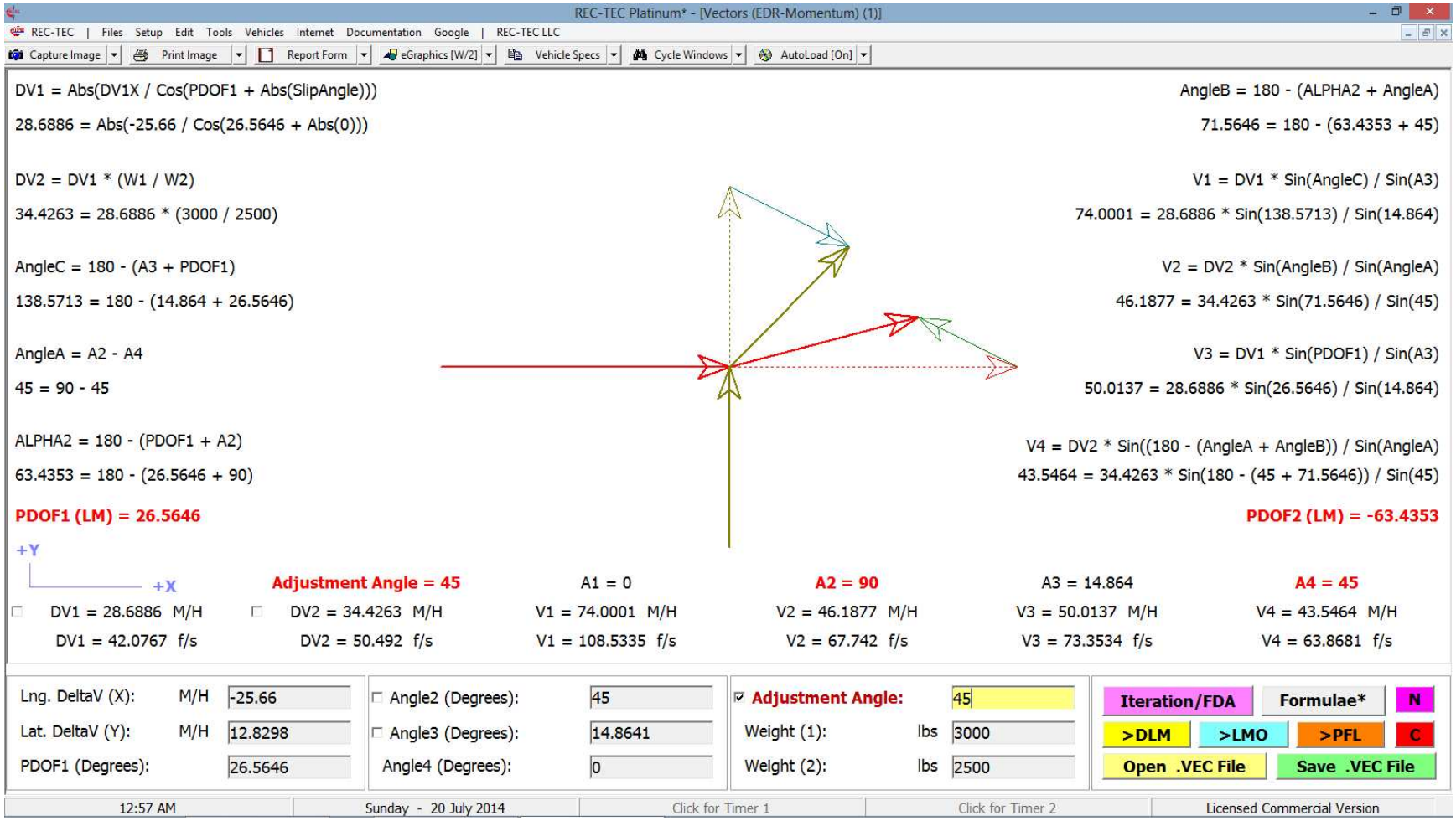
Summary:

So what is the point?

There is more than one solution that is mathematically correct, but there is only one solution that matches what actually happened. While the speed differences for Vehicle 2 at impact were minimal in these two problems, let's look at one more scenario.

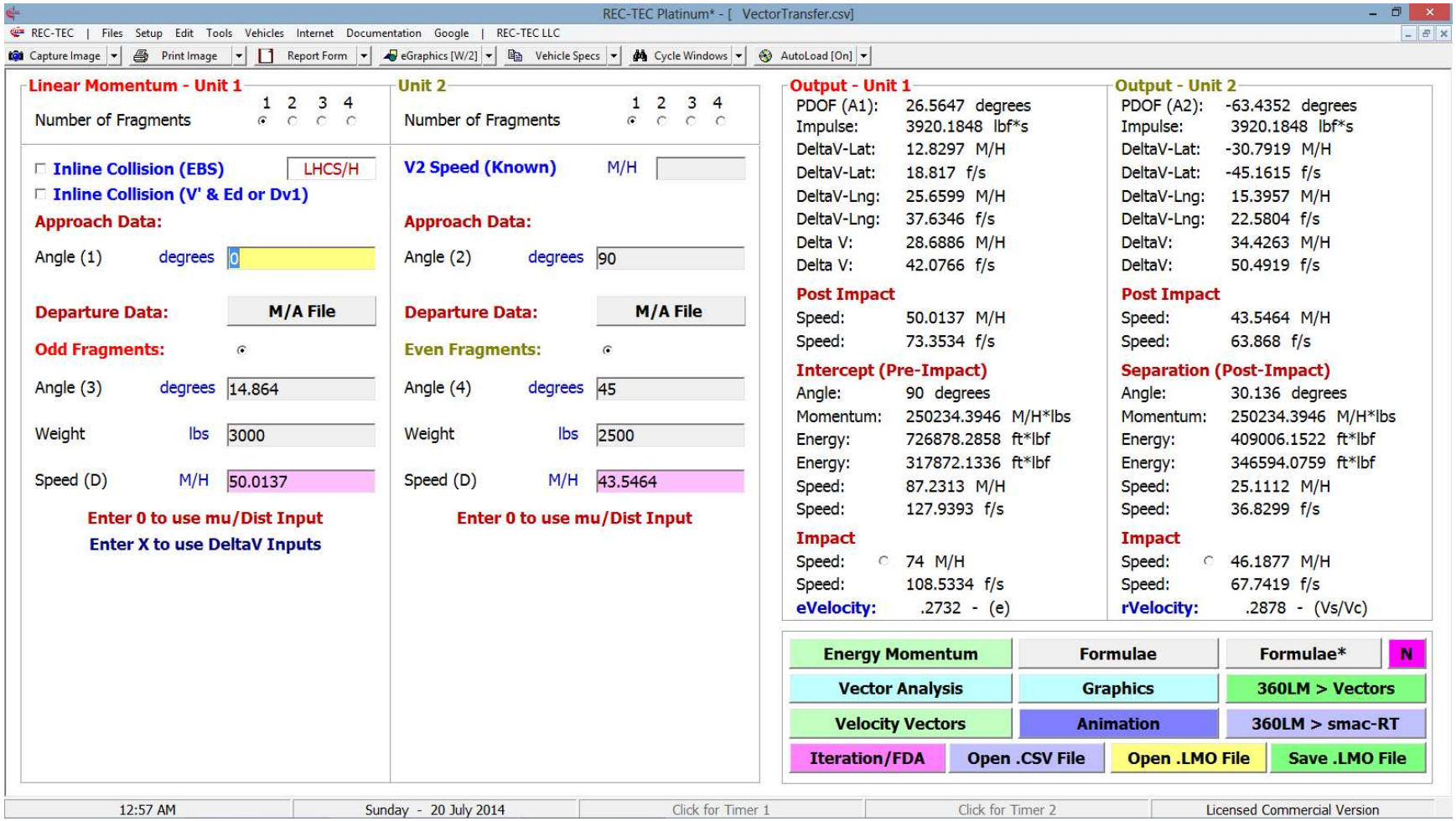
Change the Adjustment Angle to 45.

Figure 50: Vectors (EDR-Momentum) Solution - V1 Speed Input with Adjustment Angle = 45 degrees



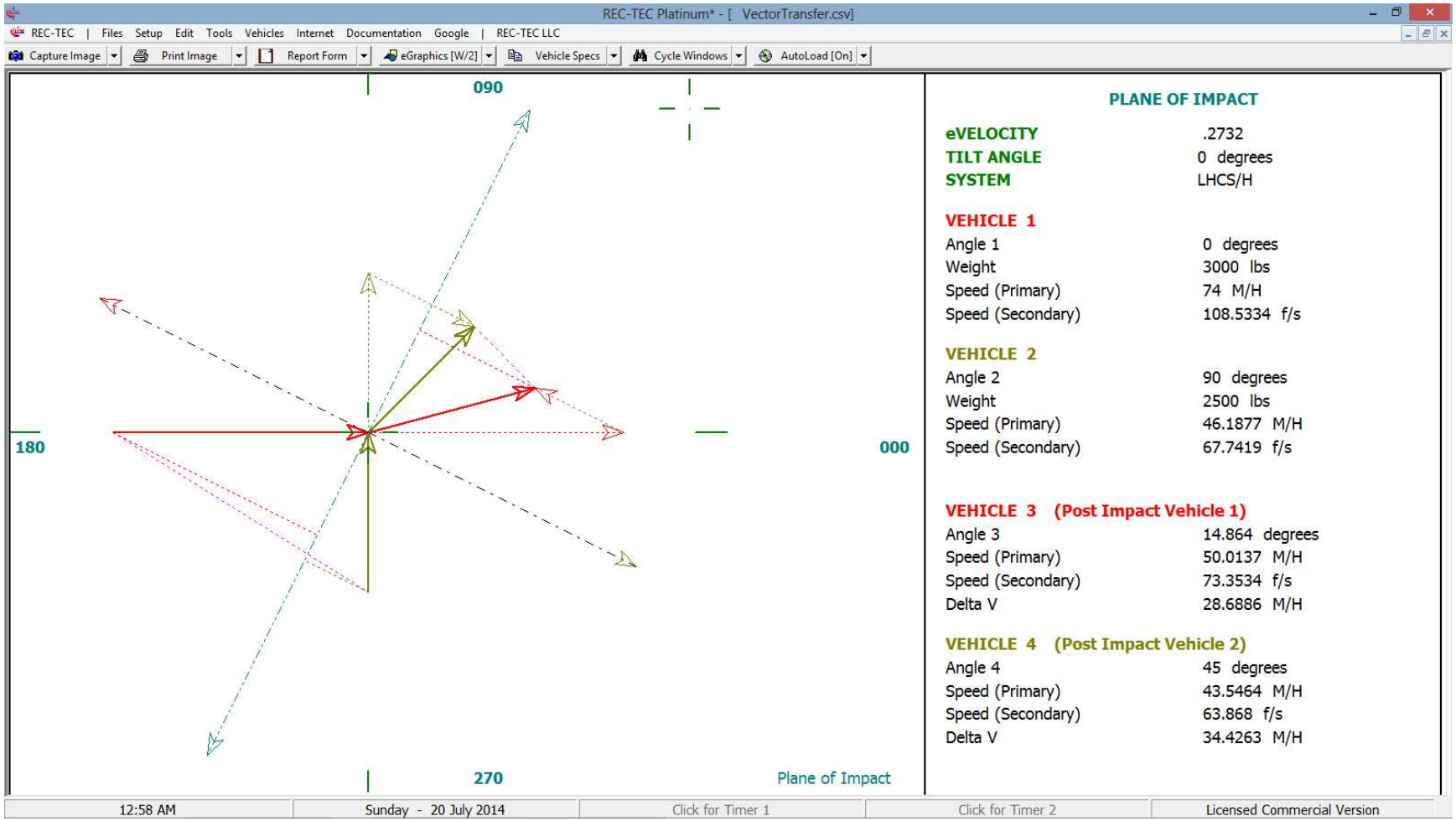
Transfer this to 360 Linear Momentum.

Figure 51: Vector Solution transferred to 360 Linear Momentum



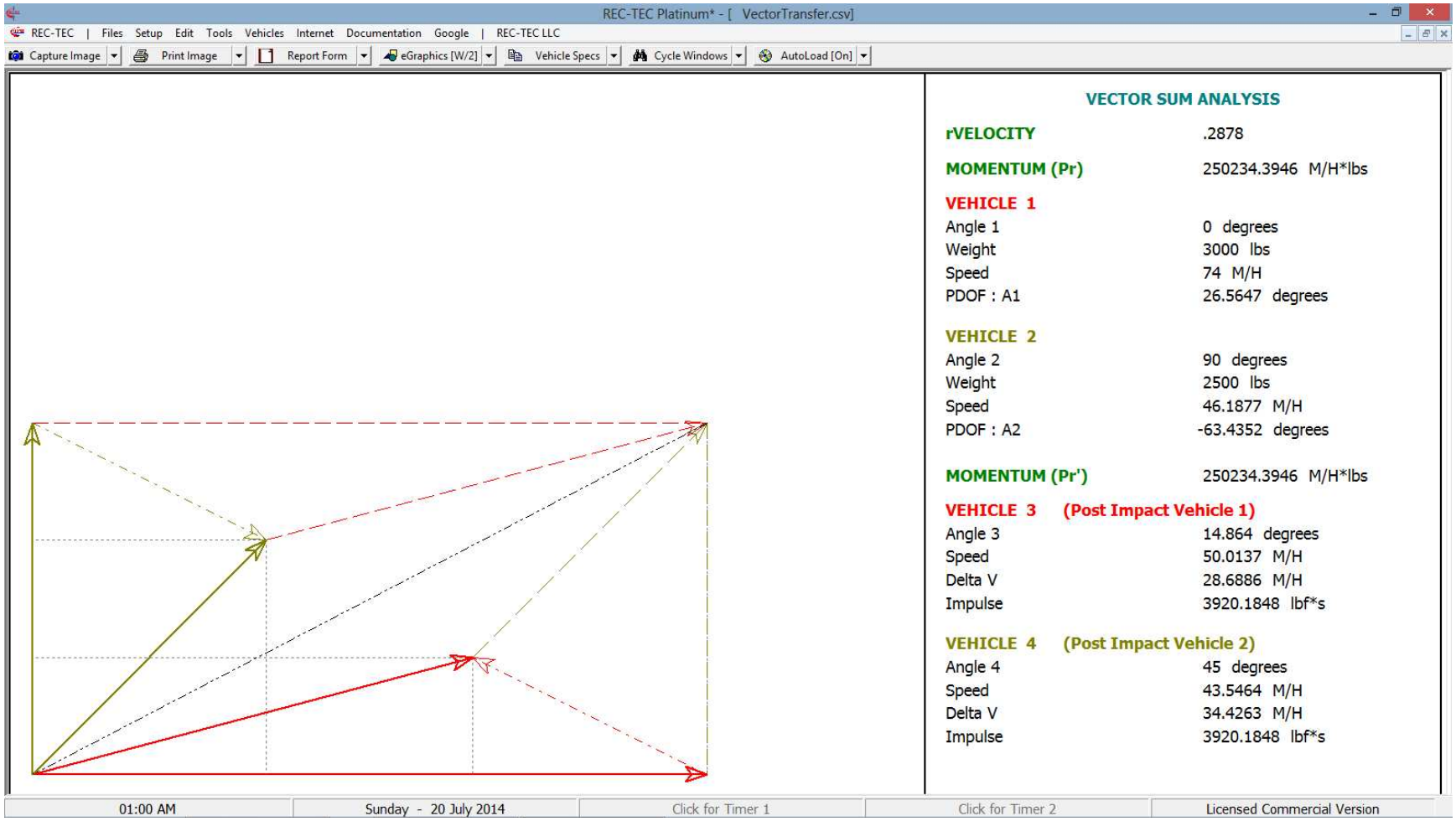
Now Select Graphics. Right click on the Mouse three times.

Figure 52: Vector Solution transferred to 360 Linear Momentum Graphics



Hit the Esc Key and Select Vector Analysis.

Figure 53: Vector Solution transferred to 360 Linear Momentum Vector Analysis



The Resultant Vectors overlap and meet at a common point so it looks OK, but **V2** has gone from **21 M/H** to over **46 M/H**. That could certainly be enough to make a huge difference, both in a Civil Venue and in a Criminal one.

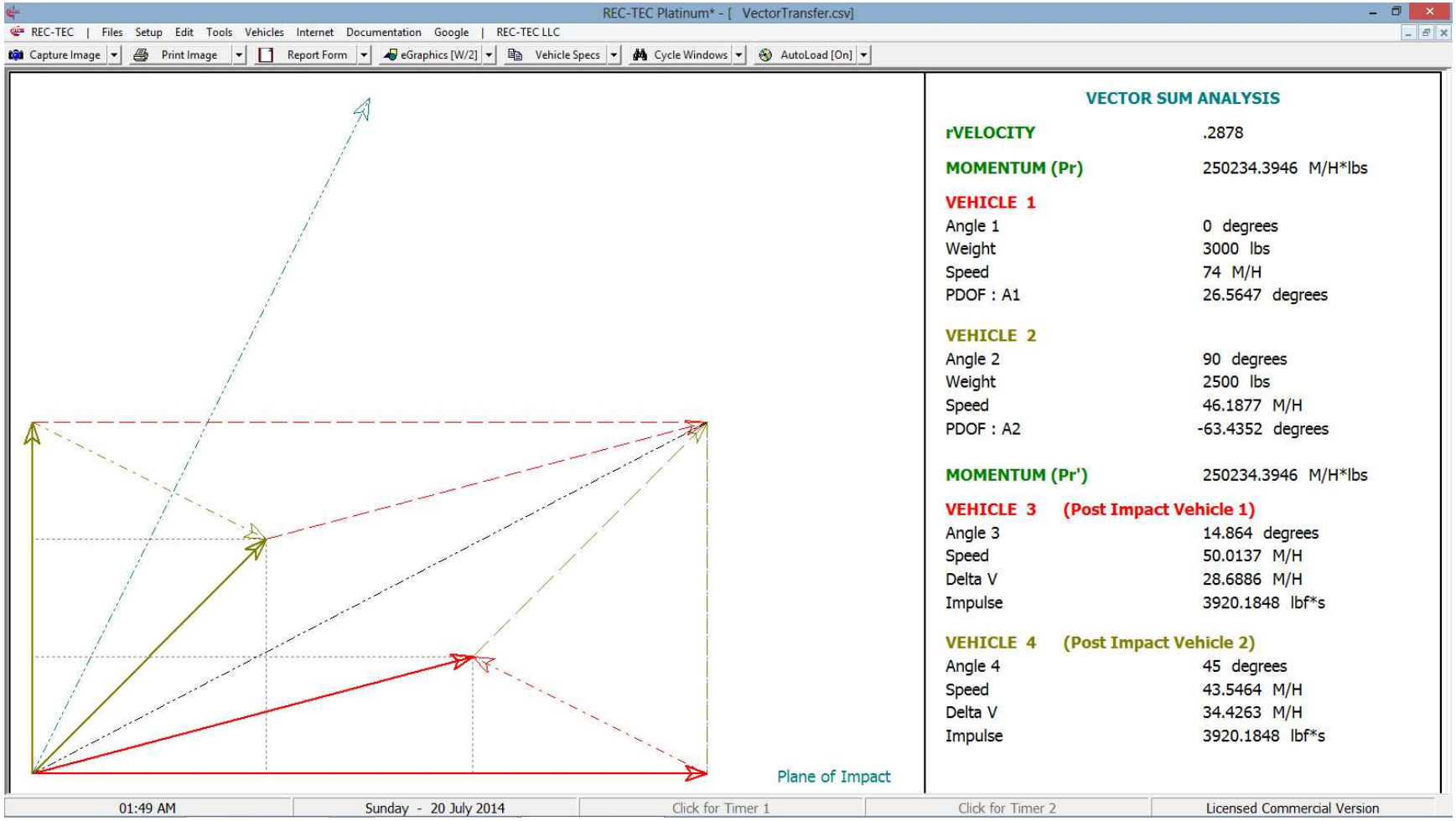
That is the point!

A Final Curiosity:

There is something about Figure 52 that is curious. Compare it to Figure 5 on page 7. The long blue line (dot-dash-dot pattern) with arrowheads at both ends passing through the collision point is the Plane of Impact. The vehicles should bounce off of that line similar to a ball bouncing off of the floor. It is almost like a mirror, reflecting beams of light off of a mirror. While not perfect analogies, there are shared qualities. Earlier it was commented that the centers of mass crossed each other, which is acceptable, and why the mirror analogy is not perfect. Still, there is something unsettling about that figure that cries out for additional scrutiny.

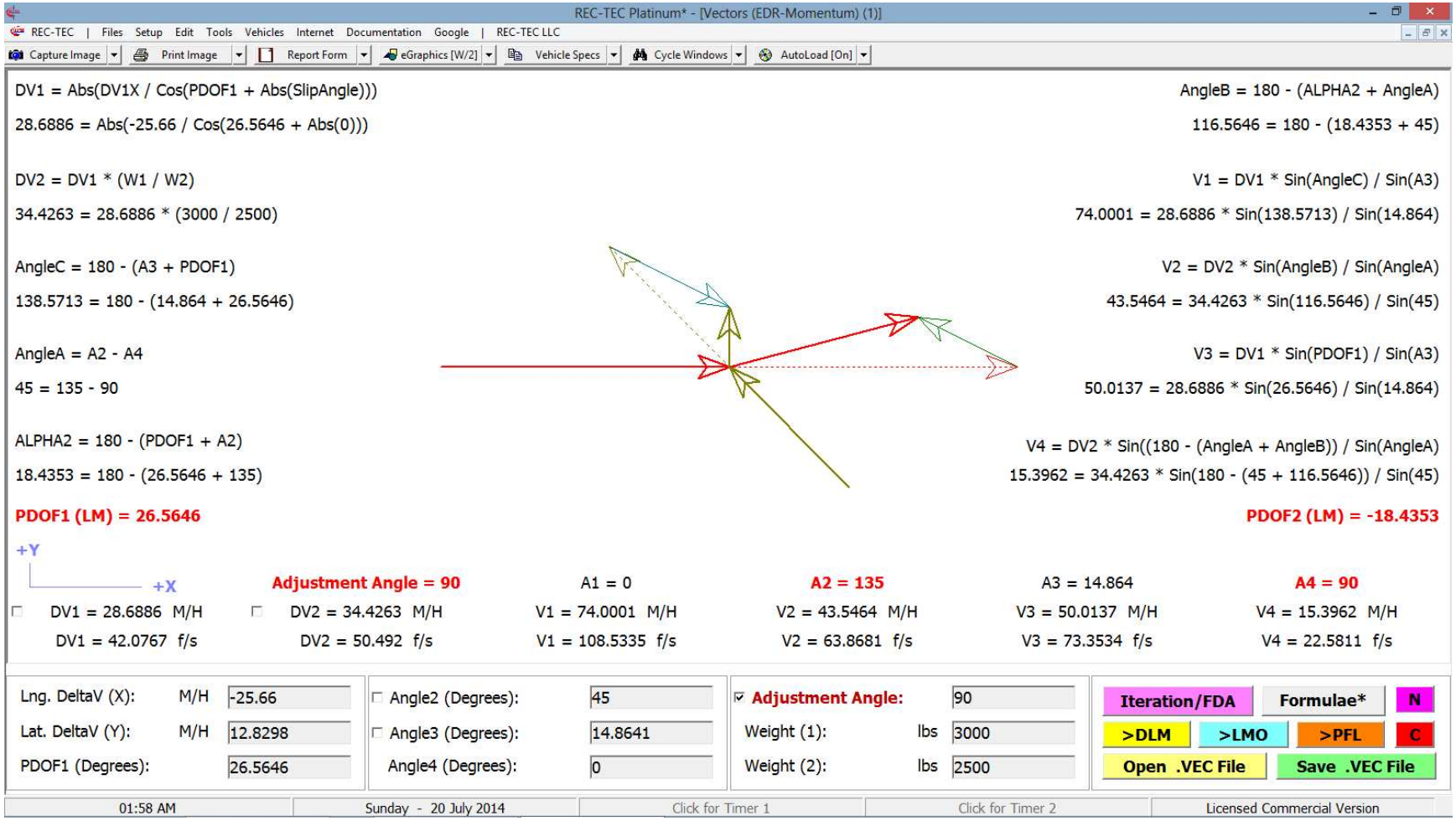
The Plane of Impact can be added to the Vector Sun Analysis Graphic, and that is the ideal place to start. With Vector Analysis graphic on the screen, right click on the drawing. The Plane of Impact is now shown on the screen.

Figure 54: Vector Solution transferred to 360 Linear Momentum Vector Analysis - Plane of Impact



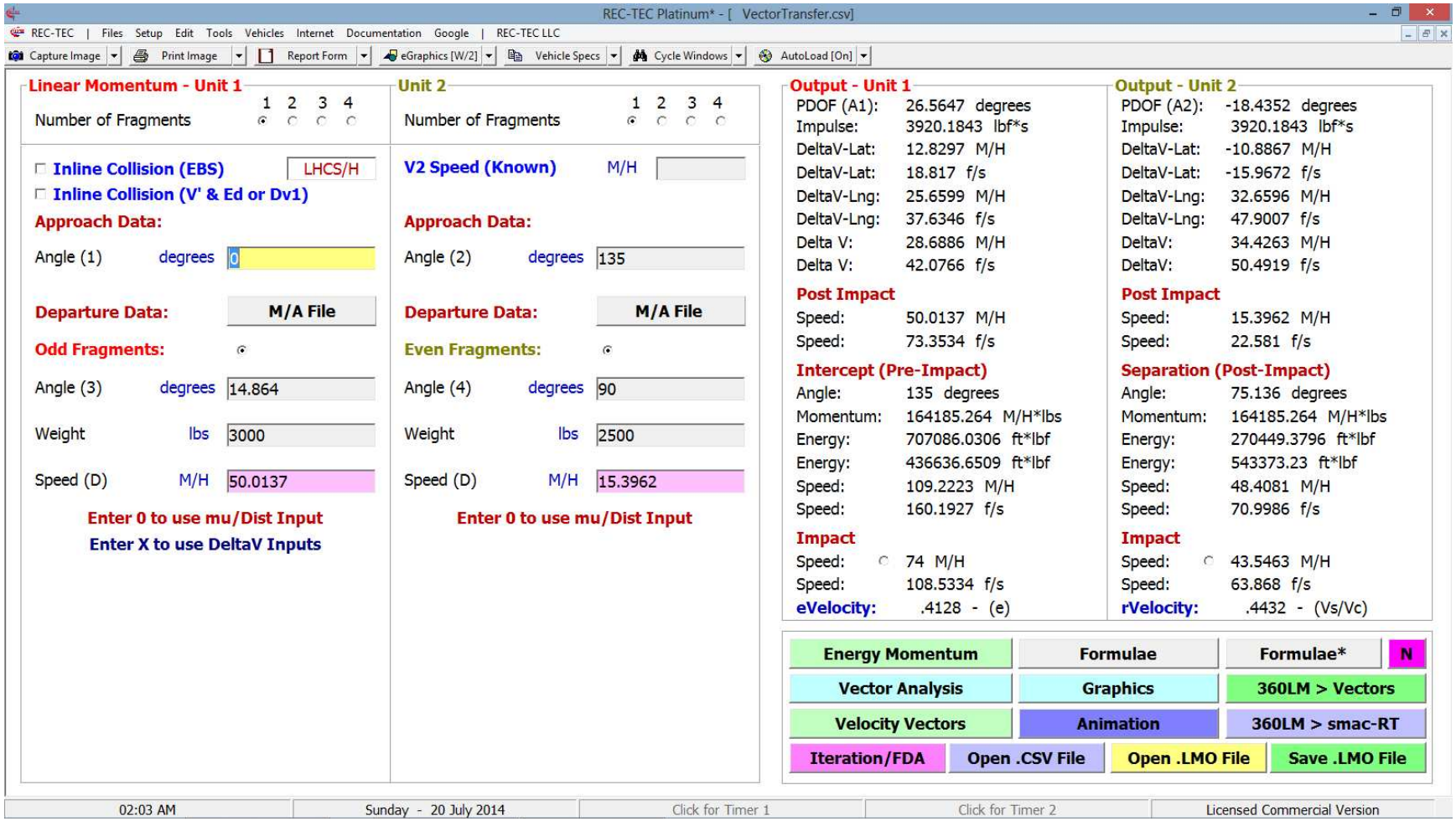
Go back to the Vector Module (Use the X in the small light colored box near the upper right of the REC-TEC Window) and change the Adjustment angle to 90 degrees.

Figure 55: Vectors (EDR-Momentum) Solution - V1 Speed Input with Adjustment Angle = 90 degrees



Transfer this data to Linear Momentum using the light blue >LMO button.

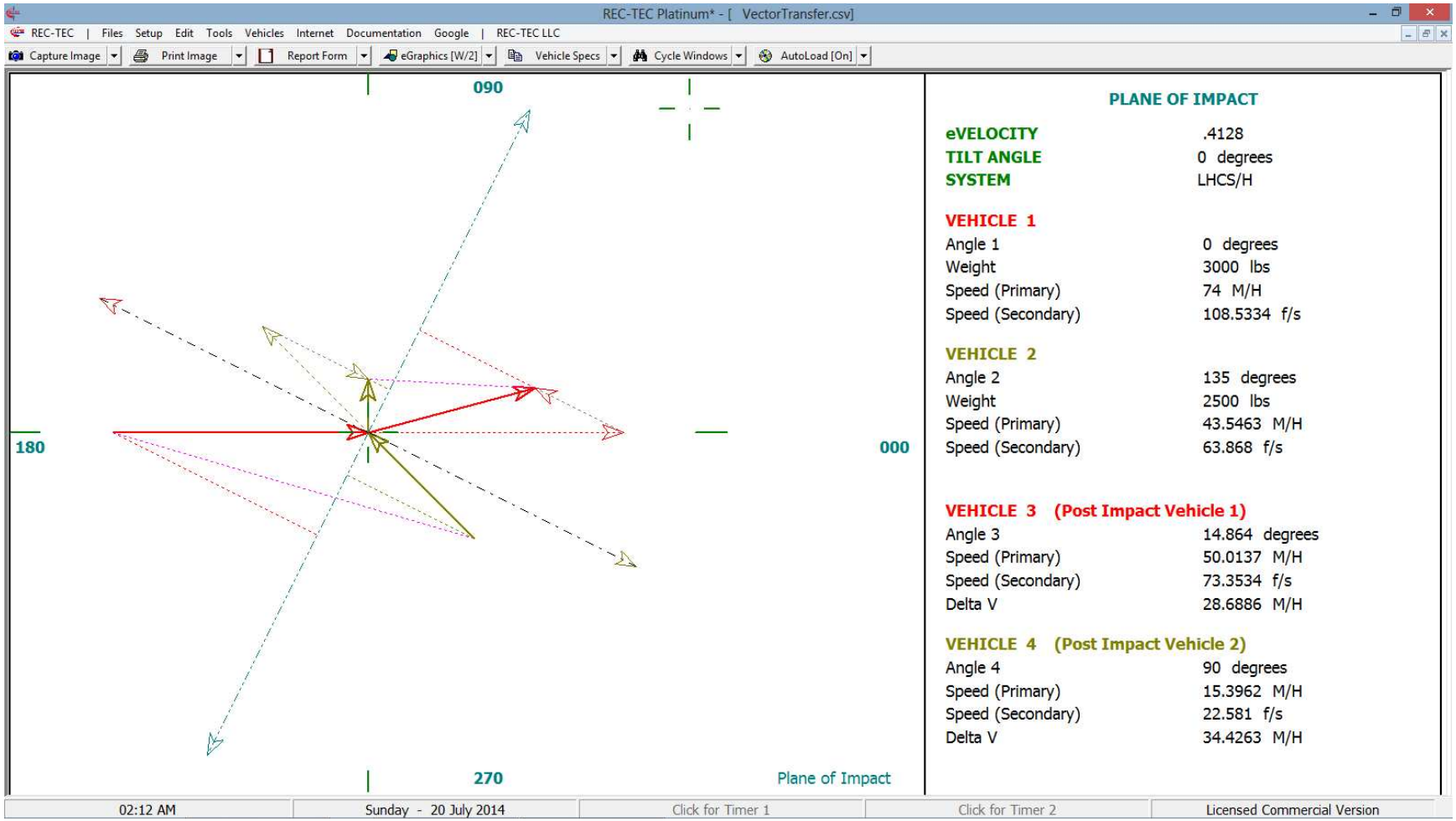
Figure 56: Vector Solution transferred to 360 Linear Momentum



Nothing Remarkable. **A2** is **135** degrees and **V2** has actually dropped to **43.5 M/H**.

Select Graphics and Right Click on the graphics 3 times.

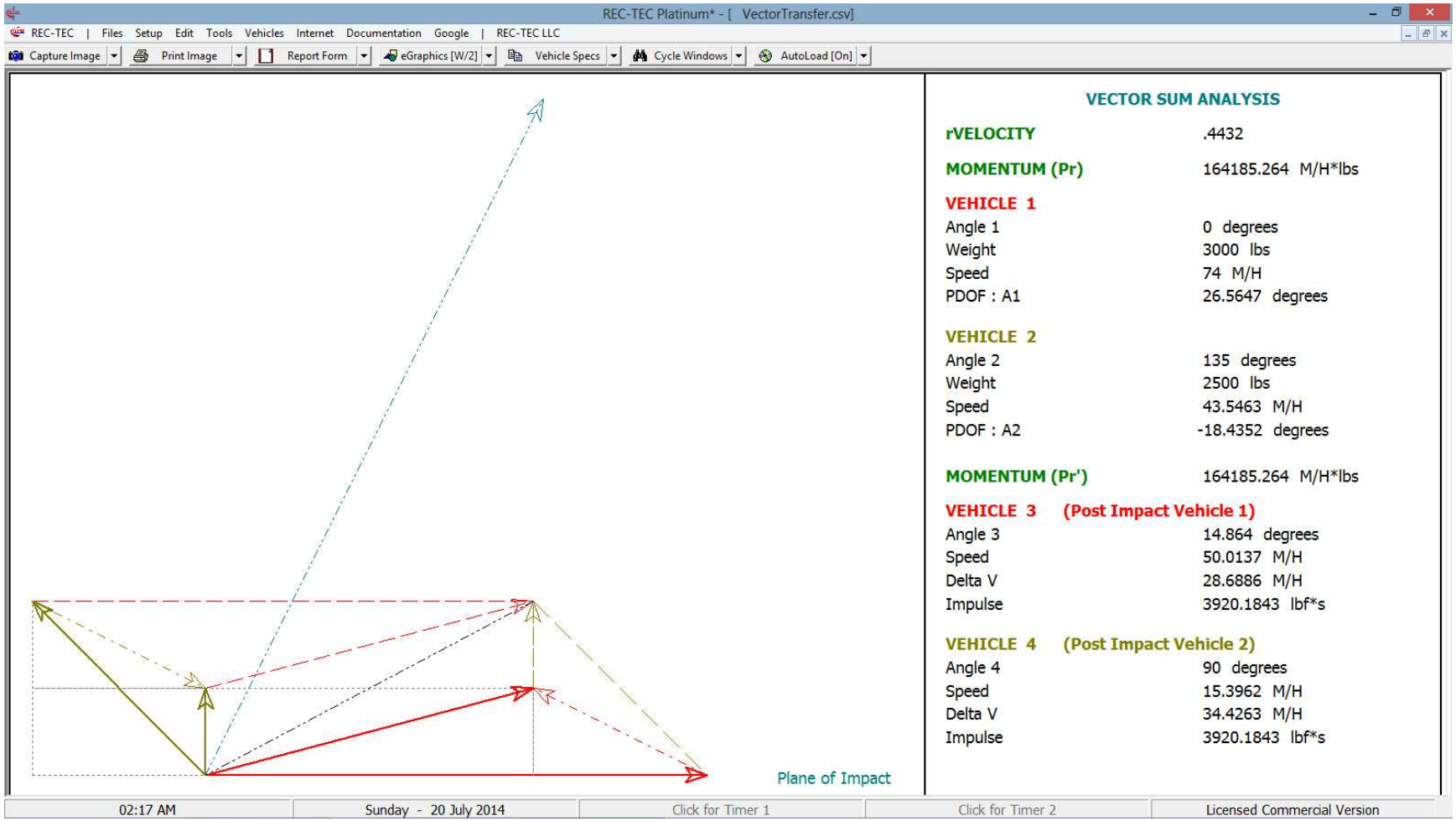
Figure 57: Vector Solution transferred to 360 Linear Momentum Graphics - Plane of Impact



The Plane of Impact is now between the post impact Vectors.

Return to the LM interface screen (Press Esc) and Select Vector Analysis. Right Click on the graphics.

Figure 58: Vector Solution transferred to 360 Linear Momentum Vector Analysis - Plane of Impact



Same results shown here, the Plane of Impact is between the departure vectors.

Conclusion:

What implications does this Plane of Impact curiosity have? Curiosity, yes, but does it rise beyond that level? Are there serious ramifications? What, if anything, does it imply?

That is a very interesting question.

Index of Figures: